Population Density Techniques for Modeling Neural Populations

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Neural circuits are often modeled in terms of populations: more or less homogeneous groups of neurons. It is assumed that each neuron experiences unique spike trains, but the statistical distributions that generate these spike trains are identical for all neurons. We assume that the populations are large, and ignore finite size effects. The basis for population density techniques is a point neuron model:

$$\tau \frac{d\vec{V}}{dt} = \vec{F}(\vec{V}) \tag{1}$$

with τ the membrane time constant in s, $\vec{V}$ the neural state and $\vec{F}(\vec{V})$ the model in question. Leaky-integrate-and-fire (LIF) neurons, quadratric-integrate-and-fire (QIF) neurons are example of one dimensional (1D) models. Conductance-based models have a more complex state space, but overall are all of the type of Eq. 1. Population density techniques (PDTs) assume that a population can be described in terms of a density function $\rho(\vec{V})$, where $\rho(\vec{V})d\vec{V}$ is the fraction of neurons in the population with state $\vec{V}$ in $d\vec{V}$. The evolution of this density is determined by a combination of synaptic input and the dynamics of the individual neuron, Eq. 1.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \vec{v}} \cdot \left(\frac{\vec{F}\rho}{\tau}\right) = \int_M d\vec{w} \left\{ W(\vec{v} \mid \vec{w})\rho(\vec{w}) - W(\vec{w} \mid \vec{v})\rho(\vec{v}) \right\} \tag{2}$$

This equation is often reduced to a Fokker-Planck equation, and even when not, numerical solutions often assume smoothness of the density profile. We show that the method of characteristics can be used to transform Eq. 2 into the Master equation of the synaptic input process. In a coordinate system that co-moves with the neural dynamics only the solution of the Master equation governing the noise is required, a considerable simplification. The gradient term of Eq. 2 is not present, removing the need for a diffusion approximation and allowing novel applications, e.g. a coherently spiking population represented by a travelling delta function. We show that this population can be modeled accurately over a long time, whereas all numerical methods known so far would quickly diffuse away this peak.

We show that the topology of the solutions of neural model Eq. 1 determines the density representation: the density of non spiking neuron models like LIF neurons is fundamentally different from that of QIF neurons, which are spiking. We show that the density representation for spiking neurons is simpler. We demonstrate that a non spiking neuron model can be turned into a spiking one, by adding an artificial current, which is compensated for by renormalizing the synaptic input. By this 'current compensation method' one can switch between the topologies of the underlying neural model, and we demonstrate that the model is universally applicable for 1D models. This is important for extending the technique to higher dimensional models.

We demonstrate that the method can be used for generalized Master equations [1]: processes with a memory kernel. We demonstrate this on an example of spike trains that are governed by a gamma distribution with shape parameter 2. This is an important extension: it removes the Poisson or white noise approximations that are implicit in current formulations of the theory.

We will show examples of neural circuits modelled with this technique that demonstrate that the state of individual neurons and that of the population may be related in non obvious ways. Whilst individual neurons may spike quasi-periodically at relatively modest rates, the population can demonstrate bursting. Experimentalists would experience this as a contrast between single unit recordings and local field potential behaviour. We believe that neural mass models would have difficulty accounting for this contradictory behaviour.

References

[1] T. Hoffmann, M.A. Porter, and R. Lambiotte, Generalized master equations for non-Poisson dynamics on networks, *Phys. Rev. E 86, 046102*, 2012.