Lower Bounds in Theoretical Connectomics

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A number of efforts are currently underway to determine the connectome or parts thereof in a variety of organisms. However, there is debate in the Neuroscience community about how useful this data will prove to be, in practice. Ascertaining the detailed physiological response properties of all the neurons in these connectomes is out of the scope of such projects and indeed out of reach of current experimental technology. It is therefore unclear to what extent knowledge of the connectome alone will advance a mechanistic understanding of computation occurring in these neuronal circuits, especially when the high-level function of the said circuit is unknown. To examine these issues, we have recently developed a theory of connectomic constraints for feedforward networks [1]. Specifically, for feedforward networks equipped with neurons that obey a deterministic spiking neuron model, we asked if just by knowing the structure of a network, we can rule out computations that it could be doing, no matter what response properties each of its neurons may have. We also stipulated the need to demonstrate a network with a different structure comprising "simple" neurons that could indeed effect the computation in question. After setting up a mathematical framework within which these questions could be precisely posed, we showed results of this form (which we call *complexity results*) for certain classes of network architectures. We also proved, mathematically, that for certain other classes of network architectures, given our limited assumptions on the individual neurons, there are fundamental limits to constraints imposed by network structure alone. Our neurons were abstract mathematical objects that satisfied a small number of axioms that correspond to certain broadly obeyed properties of neurons. Complexity results, thus, were in the form of mathematical proofs that use these axioms and the structure of the network in question to demonstrate explicit spike-train to spike-train transformations that could not be effected by any network of the said structure, which could in turn be effected by a network of a different structure. Indeed, the broad program in this line of research is to start from first principles, as we have done in [1] and prove such results for networks of axiomatic neurons with progressively larger number of axioms and also generalize the theory to treat the case of recurrent networks. The idea is to eventually use this type of theory as a starting point to rule out specific computations in neural circuits for which connectomic data is available. This might then be used to formulate hypotheses about mechanistic computation in such circuits, which could be tested experimentally.

Here, we develop additional theoretical tools and notions to address these questions. The idea is to study the space of all possible spike-train to spike-train transformations. In particular, we are interested in asking how the subset of transformations spanned by networks of specific architectures can be related to subsets of the space that are characterized by particular properties of transformations. More concretely, using mathematical properties of transformations (i.e. without any reference to neurons or networks), we identify a sequence of subsets of this space, with each subset contained in the subsequent one in the sequence. We call this sequence of subsets a *Transformation Hierarchy*. This allows us to relate sets in a transformation hierarchy to sets of transformations spanned by specific network architectures, which we call the *Complexity Classes* of those architectures. We do this by finding the "smallest" set in the hierarchy that contains the complexity class in question as a subset. This set is called the *Hierarchy Class* of the architecture with respect to the said transformation hierarchy. Even if we cannot establish the hierarchy class of a given architecture with respect to a hierarchy, proving bounds[1] on them might offer insight. A set in a hierarchy is an *upper bound* on a hierarchy class if it contains the hierarchy class as a subset. Likewise, a set in a hierarchy is a *lower bound* on a hierarchy class if the hierarchy class contains the set as a subset. As an application of these notions, we have constructed an explicit class of transformation hierarchies. For every set (starting from a certain set) in each such hierarchy, we demonstrate network architectures, for which the set in question is a lower bound on the hierarchy class of the said network architecture. We will skip a more detailed exposition of these results here for want of space.

There are at least two reasons for relating complexity classes to transformation hierarchies. First, complexity classes themselves seem to be hard to characterize succinctly in terms of properties of transformations they contain. Instead, we try to understand complexity classes of network architectures relative to these sets in the hierarchy which are easier to characterize using mathematical properties of transformations. The second reason for this approach is that it provides us another – and a possibly more wholesale – way to prove complexity results via bounds on the corresponding hierarchy classes.

References

[1] V. Ramaswamy, A. Banerjee. Connectomic Constraints on Computation in Feedforward Networks of Spiking Neurons, *Journal of Computational Neuroscience* 37 (2) pp. 209-228, 2014.

[1]Formally, the bounds are with respect to the partial ordering induced by set inclusion.