A numerical method for stochastic travelling waves in neural tissue

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In [2] a method was introduced for computing travelling wave solutions to stochastically forced PDEs. A key numerical example was the stochastic Nagumo equation. This technique was further used to examine Baer-Rinzel models of spiny dendritic tissue, see for example [1]. We introduce the basic numerical method for stochastic travelling waves.

Instead of applying the method to PDE based models we examine its application to 1D and 2D neural field equations in the presence of noise such as, for example, [3, 4]. In its simplest form we consider the numerical approximation of the stochastic differential algebraic system of the form

$$\begin{aligned} du &= [-u + K(u) + \lambda(t)u_x]\,dt + u \circ dW, \quad u(0) = u^0 \\ 0 &= \langle \hat{u}_x, u - \hat{u} \rangle \end{aligned}$$

for u and for stochastic "instantaneous" wavespeed λ. K is the operator arising from $\int_{-\infty}^{\infty} w(x-y) f(u(y,t)) dy$ and $\hat{u}$ is a fixed reference function. This constraint originates from fixing minimizing the L^2 distance of u to the reference function. We briefly discuss an improved collocation discretization for the evolution of the integral equation. We then consider the case where the wave is frozen. We also illustrate that the numerical solution of stochastic waved by freezing is compatible with the multilevel MC method.

References

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