Can we hear the shape of a brain ? Spectral analysis of cortical anatomy.

Julien Lefèvre, Aix-Marseille Université; CNRS LSIS UMR7296 ; INT UMR 7289, CNRS, Marseille; France
David Germanaud, APHP, Hôpital Robert Debré, DHU PROTECT, Paris; CEA, Neurospin, UNIACT, Gif sur Yvette
julien.lefevre@univ-amu.fr, david.germanaud@rdb.aphp.fr

Our work falls within the general context of computational anatomy whose foundations have been given more than fifteen years ago [1]. Contrary to common approaches that aim at registering shapes, in particular through diffeomorphisms, we propose descriptors of individual shapes with several applications for the understanding of brain anatomy variability or morphogenesis. Our abstract is divided in two parts: first we review results by our group to justify the interest in spectral analysis and next we expose new mathematical results that comes from data observation.

Following the celebrated article by Kac [2], we consider spectral analysis as a possible way to describe the cortical morphology, modelled as a closed surface of genus 0 and denoted $\mathcal{M}$. Briefly, our objects of interest are the eigenfunctions Φ_i and eigenvalues $0 = \lambda_0 < \lambda_1 < ... < \lambda_i < ...$ of the Laplace-Beltrami Operator $-\Delta_{\mathcal{M}}$ defined as a generalization of the Laplacian to any function $f : L^2(\mathcal{M}) \to \mathbb{R}$. In [3] we used the eigenfunctions as a basis to decompose a spatial proxy of the cortical folding pattern, in order to characterize the variability of the brain folding across a population of healthy subjects. Such an approach has also been applied to characterize pathologies such as microcephaly, a severe anomaly of brain development [4] and promissing work is still in progress for the characterisation of the developping brain. Nevertheless this approach remains classical in the sense as it generalizes Fourier analysis on a non flat surface but there is also a large amount of information related to the shape contained directly in the sequences λ_i and Φ. Since the initial question by Kac, counter-examples have revealed that there exist different surfaces that share the same spectrum λ_i [5]. That is why we proposed to study directly the spatial information of the eigenfunctions Φ_i. By using only very few low-frequency spatial modes we have observed *a priori* unexpected correlates between regions obtained through a spectral clustering and the traditional segmentation of the brain in lobes [6]. This astonishing result suggests that the geometry of the brain, at a global scale at least, could have strong relationships with its function.

The study of low-frequency eigenfunctions of the cortical surface has led us to supplementary empirical observation whose mathematical foundations are not clear at the moment. Given a function Φ, we denote $N(\Phi)$ the nodal set, i.e. the set of points where Φ vanishes. A nodal domain is a connected component of the complementary of $N(\Phi)$. Courant's nodal theorem states that the number of nodal domains for Φ_i is bounded between 2 and $i+1$. Numerical computations and visualization of the three first non trivial eigenfunction Φ_1, Φ_2, Φ_3 revealed that they had systematically two nodal domains. It suggests quite naturally to consider the following mapping $m : \mathcal{M} \to \mathbb{S}^2$ defined by:

$$m(p) := \left(\sqrt{\Phi_1(p)^2 + \Phi_2(p)^2 + \Phi_3(p)^2}\right)^{-1} \left(\Phi_1(p), \Phi_2(p), \Phi_3(p)\right)$$

Note that a similar mapping, but with more eigenfunctions, was proposed in [7]. The correct definition of our mapping can be proved in the specific case where the Φ_i have only two nodal domains. Moreover we can conjecture, with possibly restrictive hypothesis, that m is a C^∞ diffeomorphism. We will present several intermediate theorems in favor of this conjecture. We will also present potential applications of this mapping such as 1) a simple and fast visualization of brain anatomy, 2) registration of different surfaces and 3) detection of abnormal cortical patterns.

This work is funded by the French National Agency for Research (ANR-12-JS03-001-01, MODEGY)

References

[1] U. Grenander, M. Miller. Computational anatomy: An emerging discipline, *Quarterly of applied mathematics,* 56 (4) pp. 617–694, 1998.

[2] M. Kac. Can one hear the shape of a drum? *American Mathematical Monthly,* 1–23, 1966.

[3] D. Germanaud, J. Lefèvre et al. Larger is twistier: Spectral Analysis of Gyrification (SPANGY) applied to adult brain size polymorphism, *Neuroimage,* 63 (3), pp. 1257-1272, 2012.

[4] D. Germanaud, J. Lefèvre et al. Simplified gyral pattern in severe developmental microcephalies? New insights from allometric modeling for spatial and spectral analysis of gyrification, *Neuroimage*, 102, 317–331, 2014.

[5] C. Gordon, D.L. Webb, S. Wolpert One cannot hear the shape of a drum, *Bulletin of the American Mathematical Society*, 27 (1), 134–138, 1992.

[6] J. Lefèvre, G. Auzias, D. Germanaud, Brain lobes revealed by spectral clustering, *IEEE ICPR*, 562–567, 2014

[7] P. Bérard, Volume des ensembles nodaux des fonctions propres du laplacien, *Séminaire de théorie spectrale et géométrie*, 3, pp.1–9, 1984.