# Neurally plausible implementation of minimum probability flow

Christopher Hillar, Redwood Center for Theoretical Neuroscience, Berkeley, CA 94720, USA
Felix Effenberger, Max-Planck-Institute for Mathematics in the Sciences, Leipzig, Germany

Inspired by ideas of J. Movellan, the authors of [SDBD11] introduced *probability flow*, an objective function to be minimized for estimating parameters in probabilistic models from data. A version for the case of the Hopfield network (HN) is the convex function $\frac{1}{|X|}\sum_{\mathbf{x}\in X}\ \sum_{\mathbf{x}'\in\mathcal{N}(\mathbf{x})}\exp\left(\frac{E_{\mathbf{x}}-E_{\mathbf{x}'}}{2}\right)$ (1), where $\mathcal{N}(\mathbf{x})$ is the Hamming ball of radius 1 around $\mathbf{x}$ (i.e. those vectors one bit flip from $\mathbf{x}$) and $X$ denotes the set of training vectors.

It can be shown that some HN has strict memories $X$ iff (1) can be arbitrarily small, motivating minimizing (1) to find Hopfield networks storing $X$ [HSK12]. A descent of the gradient of (1) given a single pattern $X = \{\mathbf{x}\}$ is called *minimum probability flow* (MPF) learning. MPF can be seen as a generalization of the classical outer product learning rule (OPR) by Taylor-expanding $e^x \approx 1$ in the following equation (2) (i.e. it can be seen as a generalization of OPR) and in contrast to other model estimation techniques (e.g. maximizing Bayesian likelihood), MPF does not involve an unwieldy partition function over the (possibly high-dimensional) set $X$.

We show in this work MPF parameter changes for a HN defined by coupling weights $J$ and thresholds $\Theta$ given $\mathbf{x} = (x_1, \ldots, x_n)$ can be written as local updates to $J$, $\Theta$: $\Delta J_{ij} \propto -x_j \Delta x_i \exp(\Delta x_i F_i/2)$ and $\Delta\theta_i \propto \Delta x_i \exp(\Delta x_i F_i/2)$ (2). We furthermore show that in this form the rule can be implemented locally in a biologically plausible way as a combination of experimentally witnessed plasticity rules, namely (Hebbian) LTP, (anti-Hebbian) LTD, homeostatic, and structural plasticity. Moreover, we explore some of the learning rule's properties during learning.

Whereas apart from few exceptions [GW13] the underlying mathematics of many plasticity rules (and their interactions) remain to be elucidated, we show here how an interplay of different forms of plasticity implements MPF for HN, a learning procedure for which we have a normative probabilistically-motivated understanding.

## References

[GW13] M. N. Galtier and G. Wainrib. A biological gradient descent for prediction through a combination of stdp and homeostatic plasticity. *Neural Computation*, 25(11):2815–2832, 2013.

[HSK12] C. Hillar, J. Sohl-Dickstein, and K. Koepsell. Efficient and optimal Little-Hopfield auto-associative memory storage using minimum probability flow. In *4th NIPS workshop on Discrete Optimization in Machine Learning (DISCML)*, 2012.

[SDBD11] J. Sohl-Dickstein, P. B. Battaglino, and M. R. DeWeese. New method for parameter estimation in probabilistic models: minimum probability flow. *Phys. Rev. Lett.*, 107(22):220601, 2011.