Inspired by ideas of J. Movellan, the authors of [SDBD11] introduced probability flow, an objective function to be minimized for estimating parameters in probabilistic models from data. A version for the case of the Hopfield network (HN) is the convex function 

\[
\frac{1}{|X|} \sum_{x \in X} \sum_{x' \in N(x)} \exp \left( \frac{E_x - E_{x'}}{2} \right)
\]  

(1), where \(N(x)\) is the Hamming ball of radius 1 around \(x\) (i.e. those vectors one bit flip from \(x\)) and \(X\) denotes the set of training vectors.

It can be shown that some HN has strict memories \(X\) iff (1) can be arbitrarily small, motivating minimizing (1) to find Hopfield networks storing \(X\) [HSK12]. A descent of the gradient of (1) given a single pattern \(X = \{x\}\) is called minimum probability flow (MPF) learning. MPF can be seen as a generalization of the classical outer product learning rule (OPR) by Taylor-expanding \(e^x \approx 1\) in the following equation (2) (i.e. it can be seen as a generalization of OPR) and in contrast to other model estimation techniques (e.g. maximizing Bayesian likelihood), MPF does not involve an unwieldy partition function over the (possibly high-dimensional) set \(X\).

We show in this work MPF parameter changes for a HN defined by coupling weights \(J\) and thresholds \(\Theta\) given \(x = (x_1, \ldots, x_n)\) can be written as local updates to \(J, \Theta: \Delta j_{ij} \propto -x_j \Delta x_i \exp(\Delta x_i F_i/2)\) and \(\Delta \theta_i \propto \Delta x_i \exp(\Delta x_i F_i/2)\) (2).

We furthermore show that in this form the rule can be implemented locally in a biologically plausible way as a combination of experimentally witnessed plasticity rules, namely (Hebbian) LTP, (anti-Hebbian) LTD, homeostatic, and structural plasticity. Moreover, we explore some of the learning rule’s properties during learning.

Whereas apart from few exceptions [GW13] the underlying mathematics of many plasticity rules (and their interactions) remain to be elucidated, we show here how an interplay of different forms of plasticity implements MPF for HN, a learning procedure for which we have a normative probabilistically-motivated understanding.

References

