Information Topology: neural dynamics and adaptation

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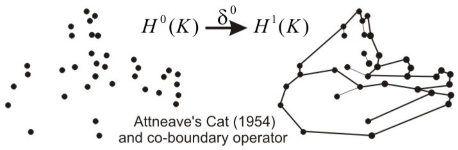
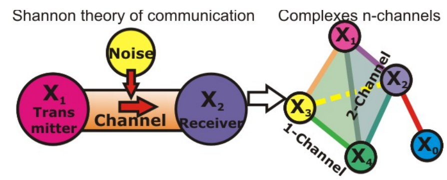
This conference is inspired by a work done in colaboration with Daniel Bennequin. The principle of efficient coding first proposed by Attneave in 1954 [1], states that the goal of sensory perception is to extract the redundancies and to find the most compressed representation of the data-environement. In his princept paper, Attneave notably claimed that any kind of symmetry and invariance are information redundancies and that Gestalt principles of perception can be defined on information theoretic ground. Information topology [2][3], beside its own mathematical and physical motivations, is an attempt to formalize this intuition, and to provide an algebraic topology framework for adaptation and learning. The purpose of this presentation is to introduce to the formalism of information topology and to





expose the application of information topology to some neuroscience problems, paradigms and data. We first describe how classical information structures are constructed and give examples of random-variables complexes. Then, two co-boundary operators, δ and δ_t, corresponding respectively to the usual non-homogenous bar construction and to topological complex, allow to define a doubled cohomology. The first theorem states that entropy is the first cohomology group. The minimal information structure such that this theorem holds, and hence such that we have a cohomology, has a fundamental interpretation in terms of complex systems self-organisation and cognition. This minimal condition is that the information structure is composed of 2 random variables with a probability space of 4 atomic events. Interpreting those two variables as classically in biology or neuroscience [4][5], as the variables associated to a (neural) system and to the environment, this condition simply state that for information to exist as a cohomological invariant it is necessary that the information structure contains at least two entities, called the system and the environment. The second theorem asserts that the mutual-information between n-tuplets of variables as defined by Hu [6] are co-cycles of the bar and topological complexes, for odd and even n respectively. Those functions quantify all the statistical dependencies between n-tuplets of variables, and display the remarkable property that they can be negative for $n > 2$, which is equivalent to synergy [7]. We show that in some cases, they can be interpreted as Homotopical invariant such as Milnor-Massey invariants for links, hence providing a definition and quantification of emmergent collective interaction. We show several theorems caracterizing this negativity, notably negativity of n-mutual-information implies that the variable do not form a Markov chain. All this give a generalisation of Shannon communication theory to arbitrary numerous emitter and receiver, defining n-channels of communication on random variable complexes. We then describe the extention of information formalism on tree structures [3], a quasi-operad model, by considering the category of ordered partitions. It provides a non-commutative generalization of information functions and a cohomological formalism defined on finite rooted trees decorated by random variables, where edges represent the values taken by the random variable. This extension gives new insights in optimal coding, Minimum Description Length, and renormalisation but also on the biological side on differentiation processes. We present the different ways of estimating those informations functions on neural data like extracellular multiple recordings, EEG, MEG or fMRI recordings. Thanks to the finiteness of the axiomatization of information [2][3][8], we propose to revisit and overcome the sampling problem originally raised by Strong and al [9], and but also to overcome also metric assumptions usually done by spin lattice models. On this bases, we propose a method extending the probabilistic and information framework proposed by Sharpee and al [10] to the full range of mutual information orders, that characterizes and estimates the topological structure of Spatio-Temporal Receptive fields. As an introduction to topological and adaptation learning principles, we reproduce the original proposition by Poincare to account for adaptation and Weber-Fechner law as a direct consequence of Homology. We conjecture, following Jaynes maximum entropy principle, that cortical topology evolves during a learning-conditioning task differentially for the odd and even n-mutual-information. We give an interpretation of learning processes in term of fluctuation-dissipation theorem using Parondo and al expression [11]. This allows to define the classical irreversibility of thermodynamic as a causal relation. Finally, we will review the information integration models of consciousness proposed by Tononi [12] and will show how information topology formalise and generalise this view. Notably, we will propose two possible analytic axiomatization arising from information topology, either grounded on Cohomology axiomatisation following Eilenberg-Maclane, or grounded on Homotopy and quasi-operad axiomatization. We hence recover a modern formal expression of the original monadic point of view that Leibniz developed in his qualitative geometry and analysis situ.

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