

# Spatio-temporal geometry of motion perception in primary visual cortex

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The receptive field of visually responsive neurons in primary visual cortex is inherently a spatiotemporal entity, as shown e.g. by works of G. De Angelis et al. Its classical linear behavior can be characterized as a three dimensional Gabor filter, selective to locally oriented uniformly moving stimuli, and the distribution of such spatio-temporal receptive fields is optimally shaped to minimize uncertainty in the estimation of wavefronts normal velocity [1].

This family of receptive profiles is then able to represent moving visual stimuli on a five dimensional manifold  $\mathcal{M}$  of spatial position and activation times, together with the local features of orientations and velocities. The point trajectories and moving contours of moving stimuli in the visual space define two dimensional level sets that are lifted to two dimensional submanifolds, and their tangent spaces define a contact bundle on  $\mathcal{M}$ . By extending works of D. Mumford to this higher dimensional geometry, one can define a signal propagation process constituted by a diffusion on the feature variables with drift on the spatio-temporal variables, performed along the admissible directions of the contact structure. The associated density equation is a subelliptic Fokker-Planck equation, whose fundamental solution has been used to model a spatio-temporal connectivity in primary visual cortex [2]. The resulting connectivity kernels allow to set up a neural field equation on the feature space  $\mathcal{M}$  whose dynamics is able to reproduce neurophysiological findings on cortical responses of motion detection, such as those described in W. Wu, P. Tiesinga, T. Tucker, S. Mitroff and D. Fitzpatrick, *Dynamics of Population Response to Changes of Motion Direction in Primary Visual Cortex*, J. Neurosci. 2011, showing evidence of a nonlinear dynamic preactivation behavior. From the cognitive point of view, this result shows a mechanism implementing the Gestalt principle known as “good continuation” of spatio-temporal visual stimuli, providing at the same time a correlation with neural activity.

Geometric connectivities modeled with stochastic kernels define a sub-Riemannian metric on  $\mathcal{M}$ , which measures the distance between two points in terms of the length of a spatio-temporal trajectory that can connect them along the contact structure. By performing a Gabor analysis of moving visual stimuli, one can then obtain a directed graph whose points are lifting on  $\mathcal{M}$  of moving stimuli and whose edges are given by the obtained distance. By performing a spectral analysis of such a graph, one can obtain a partition in terms of the eigenvectors with higher eigenvalue as a normalized spectral clustering procedure. Such a partition selects the best accessed areas by the introduced geometric diffusion, hence defining clusters in the visual stimulus as the sets of points that are mutually closer and relatively distant from the rest of the stimulus. The result is the emergence of perceptual units associated to contours in motion [3]. Physiological plausibility of this spectral analysis is associated to the normalization of connectivities and to the stability and bifurcation structure of the neural field equations. On the other hand, the psychophysical mechanism associated to this geometric clustering is an implementation of the Gestalt principle known as “common fate”, since it is able to group together areas of the visual stimulus according not only to their proximity but also to their similar motion behavior.

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## References

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