Coarse grained analysis of patterned activity in a discrete-time neural network

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Certain neural systems show computation through patterned activity; persistent localised activity, in the form of *bumps* has been linked to working memory, whilst the propagation of activity in the form of waves has been associated with binocular rivalry tasks. The assumption of infinitely fast synapses allows for the replacement of firing patterns with firing rates, resulting in a neural field model that is amenable to perturbative analysis. This description of the network averages out fluctuations in both space and time ignoring these small scale effects. Our aim is to perform analysis on a network that retains these small scale effects, but whose large scale effects can be predicted in an analogous way to neural field models.

We present analysis of a network of non-locally pulse-coupled three-state neurons whose transitions are probabilistic [1], so that the network can be realised as a Markov chain. By first ignoring the stochastic effects and considering a network of infinitely many neurons, we demonstrate the existence of bump and wave states in terms of the synaptic current profile. We then go on to show how coarse-grained analysis can be used to construct bifurcation diagrams for the network when these limits are relaxed and show how these can be used to reduce the complexity of the dynamics.

Recasting the Markov chain as a dynamical system defined over sets of neurons sharing the same state, we show how to perform stability calculations so that we make concrete predictions about observed states in the network. We show how analysing the network through this lens achieves a significant reduction in the complexity of the network. This gives rise to the bifurcation diagram: Fig. 1, which shows how the characteristic width of patterned solutions varies as the coupling strength between neurons is varied.

Through coarse-grained analysis, we can connect the fine and coarse scales of our model, and show that these states persist as we move away from the limit of infinitely many neurons, showing that finite-size effects are not sufficient to disrupt these phenomena. Finally, we now reinstate the random fluctuations in the model, and we perform existence and stability calculations using the equation-free method [2]. We show that coherent network states are robust to the inclusion of these noise sources, but also find bifurcations between distinct states as the strength of the stochastic processes on the network dynamics is varied. These results show that our multi-scale approach can be used to perform analysis at a coarse scale, whilst respecting fine scale fluctuations. This can potentially provide an avenue for analysing models of biological networks that respect their underlying biophysical complexity. Future work will investigate network structures that are informed by experiments on real neural networks.

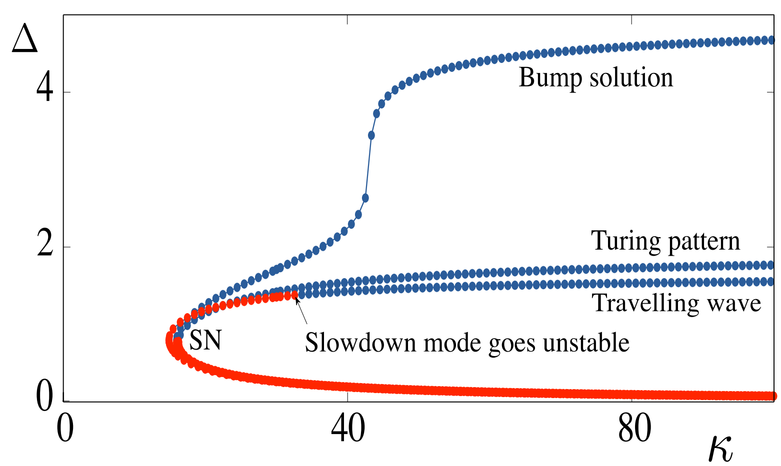

Figure 1: Bifurcation diagram for the network showing how the width of patterned solutions depends on the strength of coupling between neurons.

References

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