A new definition of the firing time for stochastic Leaky Integrate and Fire neuronal models

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Leaky Integrate and Fire models are among the most popular representation of spiking activity of a neuron as they describe in a simple way the membrane potential dynamics of a single neuron.
However the estimation of the firing threshold value presents difficulties. Indeed, the observation of experimental intracellular recordings shows that the membrane potential may cross the threshold level several times before an action potential is generated. This requires that the spiking mechanism typical of LIF models has to be changed.
We study a new definition of the firing time of a neuron. We suppose that a spike is generated if the membrane potential reaches the firing threshold and remains above this level for a sufficiently long time interval. From the biological point of view, the introduction of a time window describes the time needed for the opening of voltage dependent ion channels that are involved in the generation of the action potential.
More specifically, the new firing time is described by the random variable $H^+_{\theta,D} = \inf\{t \geq 0 | 1_{x(t)\geq\theta}(t - g_{\theta,t}) \geq D\}$, with $g_{\theta,t} = \sup\{s \leq t; x(s) = \theta\}$ and D the period that the process has to pass above the threshold θ. The properties of $H^+_{\theta,D}$ are studied for a Wiener and a Ornstein-Uhlenbeck process.
Ito excursion theory plays an essential role in this study. Specifically, we review the question of the first excursion exceeding a fixed length u of a diffusion process. Main results regarding this problem has been developed in [1],[2]. We specialize their conclusions for a Wiener and Ornstein-Uhlenbeck process.
Finally, a comparison between the classical and the new firing model is performed. The proposed spiking mechanism enhances the range of behaviors of the modeled neuron. Contrary to the LIF models, the neuron with the new firing definition behaves both as an integrator and as a coincidence detector, depending on the width of the introduced window.

References

[1] R.K. Getoor. Excursions of a Markov process, *The Annals of Probability* 7 (2) 244-266, 1979.

[2] M. Chesney, M. Jeanblanc-Picqué, M. Yor. Brownian Excursions and Parisian Barrier Options, *Advances in Applied Probability* 29 (1) pp. 165-184, 1997.