Wild dynamics in nonlinear integrate-and-fire neurons: mixed-mode bursting, spike adding and chaos.

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Nonlinear bidimensional integrate-and-fire neuron models studied by us are hybrid dynamical systems, combining continuous differential equations and discrete resets in order to model the membrane potential, the so-called adaptation variable and spikes which are defined by the divergence of the membrane potential variable to infinity. These simple models reproduce a large number of electrophysiological features displayed by real neurons, such as regular spiking and bursting. The nature of the spike pattern has been related to the orbits of a discrete map, the *adaptation map*, which was studied in several situations where it was regular. We analyse here cases where the adaptation map is discontinuous, and show that extremely interesting behaviours appear. We particularly focus on a new, very wild type of oscillations: the mixed-mode bursting, corresponding to bursts interspersed with small oscillations, which was not reported in planar hybrid systems before.

We perform rigorous mathematical analysis using methods of low-dimensional dynamics such as rotation numbers and rotation intervals of (discontinuous) interval and circle mappings. These theoretical findings are illustrated by numerical examples.

Our study applies to the wide class of spiking models, including quartic model and adaptive-exponential model.

References

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