Nonlinear noisy integrate and fire neuron models: delay and excitatory-inhibitory populations

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The Network Nonlinear Noisy Leaky Integrate and Fire (NNLIF) models for neuronal networks can be written as Fokker-Planck-Kolmogorov equations and give a rule to determine the probability to find a neuron at the potential v. One of the main parameters of the model is the connectivity of the network, which, under some assumptions, leads to a blow-up in finite time of the solution. Specifically, in [2] it has been proven that for fully excitatory networks, if the initial data is concentrated enough around the firing potential or if the connectivity parameter is big enough, there are not any global-in-time weak solutions for the system. In [3], this result was extended to a NNLIF model which includes a refractory state, which means that, after firing, neurons do not respond to any stimuli during a period of time (refractory period). In both papers, [2] and [3], a theoretical and numerical analysis of steady states for the models was developed.

The first aim of this talk is to describe the behavior of solutions of the NNLIF model, when the delay in the neuronal transmission is included. This completes the model since usually the neuronal transmission is considered to suffer a delay [1].

We focus our study in the behavior of solutions that in the NNLIF model without delay led to a blow-up in finite time. After some numerical analysis, we observe that the inclusion of a delay avoids the blow-up in finite time, as it was demonstrated in [6]. In fact, the graphic of the firing rate shows a first high increase and falls down - with some oscillations- to a steady state afterwards. The maximum of the first increase rises if the value of the delay is smaller.

In the NNLIF model which includes a refractory state and a delay sometimes a periodic behavior of the firing rate is expected [1].

The next goal of this talk is to describe results on the excitatory-inhibitory coupled NNLIF model. There we consider a neuronal network that is composed of two different populations: excitatory neurons and inhibitory neurons. In a first approach we are going to suppose that none of the populations has a transmission delay nor a refractory period.

In this model solutions can also blow-up in finite time. The blow-up phenoma here is reflected in a non-existence of a global-in-time solution for the probability density of the excitatory population. Since the equation for the probability density of the inhibitory population is coupled to the excitatory one, via the firing rate, which blows-up in finite time, its solution also fails to be global in time in that case. In [4] it was proved that in fully inhibitory networks the solutions are global in time.

Last, we analyse the number of steady states in terms of the parameter values of the model. For this excitatory-inhibitory model the parameter space is quite huge, so the observed dynamic is very rich: there are cases with one, two, three or none steady states. In fact, for the parameter space that does not give any steady state, it seems that the solutions always blow-up in finite time.

In the future we hope to be able to extend our results to the general case, which means, to the case in which the excitatory-inhibitory coupled model includes a transmission delay and a refractory state.

References

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