Elements of a finite-size ergodic theory in balanced spiking networks

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The stability of a dynamics constrains its ability to process information, a notion intended to be captured by the ergodic theory of chaos [1] and one likely to be important for neuroscience. The asynchronous and irregular network activity observed in the cortex can be produced by models in which excitatory and inhibitory inputs are balanced [2]. For negative and sharply pulsed interactions, the dynamics of these networks are linearly stable with a negative maximum Lyapunov exponent [3]. The surprising coexistence of aperiodic activity and stability was originally discovered in coupled map lattices and dubbed *stable chaos* [4]. Stability to perturbations was found to exist only up to some finite average strength beyond which the perturbed trajectory diverges exponentially [5]. This demonstrates the existence of a large, finite set of locally-attracting irregular, asynchronous spike sequences, each contained in a *flux tube* attractor basin, the latter set of which are mutually-repelling. In this contribution we address (1) where stable chaos comes from and (2) how far it extends away from idealized models remained unclear.

We answer the first question for purely inhibitory, randomly-connected and pulse-coupled networks of Leaky Integrate-and-Fire (LIF) neurons through exact, event-based simulations and analytical calculations. We first explain the spiking instability, including a derivation of the pseudo-Lyapunov exponent, which is positive. The instability's characteristics are then leveraged in a derivation of the fraction of restored perturbations as a function of the perturbation strength. The result is the q-Pochhammer symbol from the theory of partitions, reflecting the set of all causal paths through the network beginning from the time of the perturbation. The characteristic scale of the derived function gives the average size of the cross sections of flux tubes, and thus, of the attractor basins of this dynamics. The parameter dependence of this characteristic scale makes clear for the first time the contributions from the coupling, connectivity, and single neuron properties. In the context of a decoding partition of the phase space, the non-zero entropy produced by the spiking instability, which is not captured by conventional ergodic theory, can then be estimated.

To address the second question, that of the robustness of this attractor geometry, we lifted two idealizations of the LIF model by smoothing the discontinuous threshold and the discontinuous pulse-coupling, respectively. Stable chaos persists in both of these extensions up to a finite amount of smoothing beyond which conventional chaos takes hold. For smoothed pulse-coupling, stable chaos persists in the infinite network size limit, emerging from conventional chaos with the speed of the synaptic interactions and with the average size of the tube cross section growing quadratically away from this transition.

The work in this contribution provides a foundation for understanding the phase space geometry and information processing capacity of networks in the fast-synapse, fast-action potential onset, and inhibition-dominated regime where the existence of finite-sized instabilities demand extensions of the ideas from the conventional ergodic of chaos.

References

[1] D. Ruelle, Chaotic evolution and strange attractors, Cambridge University Press, 1989.

[2] C. van Vreeswijk, H. Sompolinsky, Chaos in neuronal networks with balanced excitatory and inhibitory activity, Science 274, pp. 1724-1726 ,1996.

[3] R. Zillmer, et al., Desynchronization in diluted neural networks, PRE 74 (3) pp. 1–10, 2006.

[4] A. Politi, et al., Unpredictable behaviour in stable systems, EPL 22 (8), pp. 571-576, 1993.

[5] M. Monteforte, F. Wolf, Dynamic flux tubes form reservoirs of stability in neuronal circuits, PRX 2 (1), pp. 1-12, 2012.