Neural Field Models with Dendrites

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The importance that dendrites play in synaptic integration and the generation of EEG signals is well known [1], however, except for the work of Bressloff [2], to date they have been largely ignored in mathematical models of brain activity. Our work builds on the neural field model developed by Nunez [1], with the addition of a cable equation to model the dendrite.

We consider two populations of neurons, each containing excitatory pyramidal neurons and inhibitory inter-neurons, located at y and y', where x is the point along the dendrite at which the synapse occurs. The voltage $V_a(x, y, t)$ at position $x \in \mathbb{R}$ along a cable of neuron type $a \in \{P, I\}$ with somatic coordinate $y \in \mathbb{R}$ is described by the cable equation

$$\frac{\partial V_a}{\partial t} = -\frac{V_a}{\tau_a} + D_a \frac{\partial^2 V_a}{\partial x^2} + I_a(x, y, t). \tag{1}$$

Where τ_a is the membrane time-constant of the dendrite, D_a as the cable diffusion coefficient and $I_a(x, y, t)$ is the synaptic input, which we shall split into an excitatory and inhibitory part:

$$I_a = g_{aP}(V_+ - V_a) + g_{aI}(V_- - V_a), \tag{2}$$

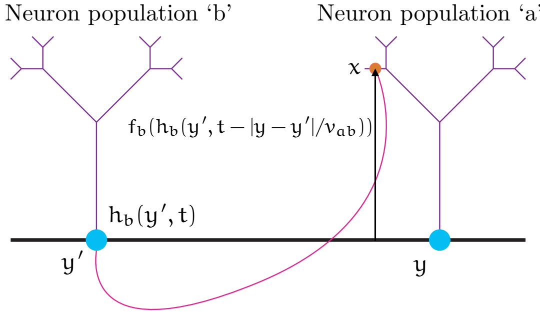

Figure 1: A schematic of the neuron-dendritic connection in our model.

where $V_+ = V_+(x)$ and $V_- = V_-(x)$ are positive and negative synaptic reversal potentials respectively, and g_{ab} is the conductance change, which can be described by the neural field equation.

$$g_{ab}(x, y, t) = g_{ab}^{\text{ext}}(x, y) + \int_{-\infty}^{t} \mathrm{d}s\, \eta_{ab}(t - s) \int_{-\infty}^{\infty} \mathrm{d}y'\, W_{ab}(x, y, y') f_b(h_b(y', s - |y - y'|/v_{ab})). \tag{3}$$

We are interested in the point at which the steady state solution of these combined equations loses stability and begins to generate wave patterns. We perform Turing instability analysis of this integral model to understand the points where these instabilities occur, following the approach of Venkov *et al* [3].

Through a series of Fourier and Fourier-Laplace transforms we obtain an equivalent PDE, or new Brainwave Equation of the model, which we use to run numerical simulations in MATLAB to produce either static patterns or travelling waves.

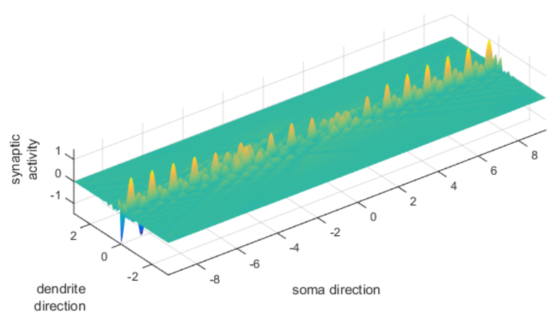

Figure 2: Example of brain waves generated by PDE simulations.

References

[1] Nunez, P.L. Srinivasan, R. Electric Fields of the Brain: The Neurophysics of EEG. Oxford Univeristy press, 2005.

[2] Bressloff, P.C. New Mechanism for Neural Pattern Formation. Physical Review Letters, 76:4644-4647, 1996.

[3] Venkov, N.A. Coombes, S. and Matthews, P.C. Dynamic instabilities in scalar neural field equations with space-dependent delays. Physica D: Nonlinear Phenomena, 232(1):1-15, 2007.

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