Neural Field Models with Dendrites

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The importance that dendrites play in synaptic integration and the generation of EEG signals is well known [1], however, except for the work of Bressloff [2], to date they have been largely ignored in mathematical models of brain activity. Our work builds on the neural field model developed by Nunez [1], with the addition of a cable equation to model the dendrite.

\[
\frac{\partial V_a}{\partial t} = -\frac{V_a}{\tau_a} + D_a \frac{\partial^2 V_a}{\partial x^2} + I_a(x, y, t). \tag{1}
\]

Where \(\tau_a\) is the membrane time-constant of the dendrite, \(D_a\) as the cable diffusion coefficient and \(I_a(x, y, t)\) is the synaptic input, which we shall split into an excitatory and inhibitory part:

\[
I_a = g_{aP}(V_+ - V_a) + g_{aI}(V_--V_a), \tag{2}
\]

where \(V_+ = V_+(x)\) and \(V_- = V_-(x)\) are positive and negative synaptic reversal potentials respectively, and \(g_{ab}\) is the conductance change, which can be described by the neural field equation.

\[
g_{ab}(x, y, t) = g_{ab}^{ext}(x, y) + \int_{-\infty}^{t} ds \eta_{ab}(t-s) \int_{-\infty}^{\infty} dy' W_{ab}(x, y, y') f_b(h_b(y', s - |y - y'|/v_{ab})). \tag{3}
\]

We are interested in the point at which the steady state solution of these combined equations loses stability and begins to generate wave patterns. We perform Turing instability analysis of this integral model to understand the points where these instabilities occur, following the approach of Venkov et al [3].

Through a series of Fourier and Fourier-Laplace transforms we obtain an equivalent PDE, or new Brainwave Equation of the model, which we use to run numerical simulations in MATLAB to produce either static patterns or travelling waves.

References


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