Finite-size effects on traveling waves in neural fields

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Neural field equations are used to describe the spatiotemporal evolution of the average activity in a network of synaptically coupled populations of neurons in the continuum limit. Since so far they have not been derived from detailed neuron models, it is in particular not clear how noise on the single neuron level translates to the population level.

The deterministic description given by the neural field equation is only accurate in the infinite population limit and the actual finite size of the populations causes deviations from the mean field behavior. Following [2] and [3], we describe the evolution of the activity in a network of populations of finite size by a Markov chain that reduces to the usual neural rate equation in the infinite population limit. The jump rates are of a different form than considered in the literature so far. They provide a new picture that seems to be more suitable when studying finite-size effects in neural fields, from a mathematical as well as from a modeling perspective, and lead to qualitatively different results.

In particular, the fluctuations around stable stationary solutions are of smaller order than previously assumed. In the case of fluctuations around stable moving patterns like traveling waves, the lowest order correction does not vanish, suggesting the movement as a main source of noise. We rigorously derive a well-posed stochastic continuum neural field equation with a noise term that can be used to study the finite-size effects on these kinds of solutions.

We also study the influence of noise on the speed of the traveling front. By dynamically adapting the speed of a deterministic reference wave profile such that the distance to the stochastic solution is minimized, we obtain an expression for the stochastic wave speed that allows for an approximate description by a stochastic differential equation to arbitrary order of the weak noise strength. Increasing the rate of adaptation to infinity, we retrieve the result as found in [4], that to first order the front wanders diffusively around its uniformly translating position.

References

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