# Neural field model with the power law response function

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Analysis and dynamical modeling of experimentally observed cortical activity is necessary to understand the nature of computations performed by the brain. A rather general class of nonlinear responses observed in numerous cortical regions [3] is referred to as "normalization". Normalization is characterized by a nonlinear integration of sensory stimuli, which can have either superlinear or sublinear behavior as a function of inputs. It is believed that such nonlinear transformation in single cortical modules can be implemented using the interaction between external inputs and network activity of surrounding neurons [1, 3, 7].

Recently, [1, 7] introduced an activity-based neural field model (1) with the goal of reproducing a class of normalization effects in sensory cortex

$$\tau_X \frac{dr_X(x,t)}{dt} + r_X(x,t) = \Big( \sum_{Y=E,I} \int_\Omega W_{XY}(x,y) r_Y(y,t) dy + g_X^{\text{ext}}(x) \Big)_+^n, \quad X = E, I. \tag{1}$$

Here, the response function $(\cdot)_+^n : \mathbb{R} \to \mathbb{R}$ is defined by $(v)_+^n = (\max\{v, 0\})^n$, $n > 1$. The firing rates $r_E(x,t)$ and $r_I(x,t)$ of excitatory and inhibitory populations depend on the space variable $x \in \Omega$ and time $t \geq 0$. Continuous functions $W_{XY} : \Omega \times \Omega \to \mathbb{R}$ represent the strength of synaptic connection between the presynaptic population of type $Y$ and the postsynaptic population of type $X$ located in $y$ and $x$, respectively. Continuous functions $g_X^{\text{ext}} : \Omega \to \mathbb{R}$ correspond to the static external feedforward inputs to excitatory and inhibitory populations and $\tau_X$ denote positive time constants. The novelty of the model (1) presented in [1, 7] in comparison to classical neural field models reviewed in [2] is the form of the response function. In contrast to the models with sigmoid, piecewise linear or Heaviside function the power law response function is unbounded, which is consistent with the observation that firing rates in visual cortex area V1 remain in the unsaturated region [6]. The idea of using the power law response function originates from [4, 5], where the authors analyze experimental results concerning activity of cells in cats visual cortex and independently deduce that the power law function can accurately reproduce the relation between the mean voltage and firing rate, both as functions of stimulus orientation and contrast. The result is based on the theoretical observation that if voltage and the firing rate functions are both multiplicatively decomposed into two functions which solely depend on orientation and contrast, then the power law is the only monotone regular function which preserves the multiplicative decomposition.

Here, we present the first analytical results concerning the stability analysis of the model (1) with $n \in \mathbb{N}$. We restrict our analysis to the functions $W_{XY}$, $g_X^{\text{ext}}$ and initial distributions $r_X^0$, which allow the following multiplicative representation of solutions $r_X(x,t) = f_X(t) h_X(x)$. We show that with the multiplicative representation we can exclude the spatial variable $x$ and reduce the model (1) to the system of ordinary differential equations for the functions $f_X$, which we investigate analytically. We prove that the reduced system can have at most 4 strictly positive fixed points for arbitrary $n > 3$ and at most 2 of equilibria are stable. We discuss, to which extent the effect of normalization can be reproduced by the reduced system.

## References

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