Canard Mediated Dynamics in a Phantom Burster

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Physiological rhythms, e.g. activity patterns of neurons, release of hormones, evolve on multiple timescales which can be modeled by using singularly perturbed, so-called slow-fast, systems. Complex behaviors of such systems result from the multiple timescales and they are often organized by underlying canard phenomena. The term canard refers to a class of limit cycles first described in van der Pol equation which stay close to an unstable slow manifold. Canards occur in singularly perturbed systems in regions of the phase space where normal hyperbolicity of the critical manifold is lost due to a bifurcation of the fast dynamics. In this context, a solution is called canard if it follows an attracting slow manifold, passes close to a bifurcation point of the fast subsystem and then follows a repelling slow manifold. In planar systems, canard cycles exist in a $O(\epsilon)$ range of control parameters where $\epsilon$ in the timescale separation parameter, whereas, in higher-dimensions, canards can exist for $O(1)$ parameter intervals. In particular, this is the case in three-dimensional systems with one fast variable and two slow variables, where canard-induced mixed-mode oscillations (MMOs) can occur. MMO refers to trajectories which consist of noticeable large and small amplitude oscillations, reappearing recurrently (periodically or not), and that can be observed in experiments and models used in various application areas. There is a recent interest in MMOs in the context of the generalized canard phenomenon in slow-fast systems, since the underlying dynamics are due to the existence of a canard structure [2]. This argument has contributed to explain the complex rhythms in neuron dynamics, e.g. excitability threshold in Hodgkin-Huxley formalism [8], firing mechanism in dopaminergic neurons [5] and subthreshold oscillations in stellate cells [7].

Collective dynamics of coupled slow-fast oscillators have a great importance in the context of physiology when microscopic and macroscopic levels can be represented as relaxation oscillators. System dynamics arise through the interactions between the intrinsic properties of the individual oscillators (multiple timescales, canard structure), the properties of connections (inhibitory, excitatory) and the network topology. Synchronization of multiple timescale systems may involve synchrony of fast timescale dynamics, spikes, and/or slow timescale dynamics, such as bursts. The case of bursting activity, that is, the alternation between slow quasi steady-state activity and fast oscillatory dynamics, introduces the question of the main mechanisms underlying transitions to synchronization and robustness of synchronization [4]. In this context, the effect of canard solutions has been considered in several aspects such as formation of clusters, synchrony and phase dynamics [3, 7].

In this presentation, we focus on the effect of canards on collective dynamics of an extended version of a neuroendocrine model which accounts for the alternating pulse and surge pattern of gonadotropin releasing hormone (GnRH) secretion introduced by [1]. The model is formed by two FitzHugh-Nagumo (FHN) oscillators that evolve on different timescales, with a feedforward coupling from the slow one (regulating system) to the fast one (secreting system). The resulting 4D model involves three different timescales. So far, global and local features of the model have been studied in the context of slow-fast dynamics and MMOs where folded singularities and associated canard trajectories have a particular importance [6]. For instance, so-called secondary canards due to a folded node are responsible for the presence of a plateau with small oscillations in the model output, after the surge and before the pulsatility resumption. We extend the model to 6D by adding one more secretor and focus on the slow-fast transitions in the presence of coupling. We explore the influence of the relationship between canard structures and coupling on patterns of synchronization and desynchronization. We propose two different sources of desynchronization, induced by canards near a folded node and canards near a folded saddle, respectively.

References