Spiral Waves : Interface Analysis in a Neural Field

Aytul Gokce Stephen Coombes Daniele Avitabile

pmxag9@nottingham.ac.uk stephen.coombes@nottingham.ac.uk daniele.avitabile@nottingham.ac.uk

Centre for Mathematical Medicine and Biology, School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK.

Spiral waves are one of the most elegant stationary (self-sustained) rotating travelling waves that settle in 2-D excitable media. Although spiral waves have been seen in many systems such as frog eggs, chicken retina, and turtle visual cortex, they have not been experimentally observed in a mammalian cortex until an experiment performed by Huang *et al.* on rodent cortical slices in 2004 [1]. From a mathematical modelling perspective, spiral waves in a non-local continuum planar model (2-D neural field model) were analysed for the first time by Laing [2]. We analyse a similar neural field model (1) and (2) given by [2] to observe rigidly rotating spiral waves, albeit for a Heaviside firing rate:

$$\frac{\partial u(\mathbf{x},t)}{\partial t} = -u(\mathbf{x},t) + \iint_{\mathcal{S}} w(|\mathbf{x}-\mathbf{x}'|)H(u(\mathbf{x}',t)-h)d\mathbf{x}' \; - a(\mathbf{x},t), \tag{1}$$

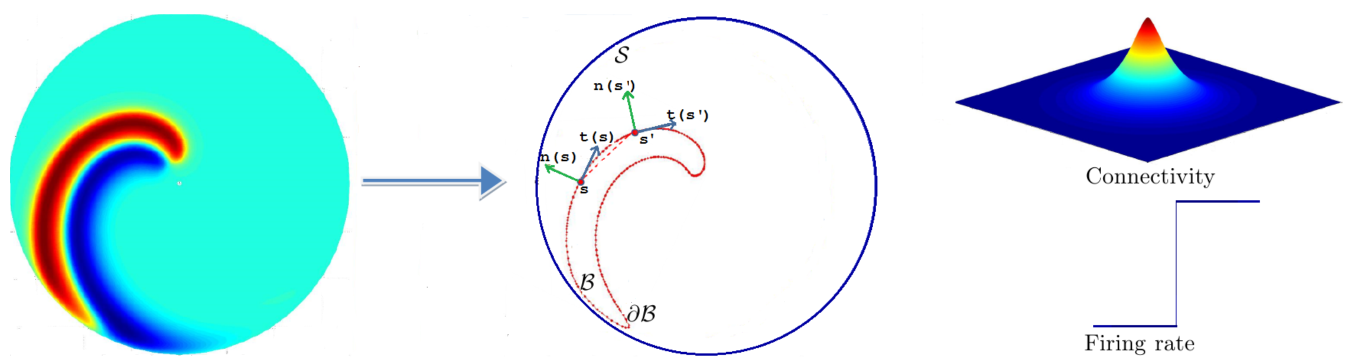
$$\tau\frac{\partial a(\mathbf{x},t)}{\partial t} = bu(\mathbf{x},t) - a(\mathbf{x},t) \tag{2}$$

posed on a disk $\mathcal{S}$ of finite radius. Spiral waves in this model are naturally defined by the border between low and high states of neural activity.

Dimensionally reduced system of equations can be derived using a recent *interface approach* [3]. Differentiating the level set $u(\mathbf{x},t) = h$ along the contour $\partial\mathcal{B}(t)$ allows us to obtain normal velocity, $c_n \equiv \mathbf{n}\cdot\frac{d\mathbf{r}}{dt} = \frac{\mathcal{F}(c_n)}{|\mathcal{F}(\mathbf{n})|}$, where $\mathbf{r}$ is a point on domain boundary. Using the Reynold's transport theorem we find (after dropping transients)

$$\mathcal{F}(z(s,t) = \int_0^t dt'\,\eta(t') \oint_{\partial\Omega(t-t')} ds'\,w(|\mathbf{r}(s,t)-\mathbf{r}(s',t')|)z(s',t-t'). \tag{3}$$

We can now analyse the interface model directly rather than the more computationally expensive space-time model defined by (1) and (2).



References

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