Spiral Waves : Interface Analysis in a Neural Field

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Spiral waves are one of the most elegant stationary (self-sustained) rotating travelling waves that settle in 2-D excitable media. Although spiral waves have been seen in many systems such as frog eggs, chicken retina, and turtle visual cortex, they have not been experimentally observed in a mammalian cortex until an experiment performed by Huang et al. on rodent cortical slices in 2004 [1]. From a mathematical modelling perspective, spiral waves in a non-local continuum planar model (2-D neural field model) were analysed for the first time by Laing [2]. We analyse a similar neural field model (1) and (2) given by [2] to observe rigidly rotating spiral waves, albeit for a Heaviside firing rate:

\[
\frac{\partial u(x, t)}{\partial t} = -u(x, t) + \int_{S} w(|x - x'|)H(u(x', t) - h)dx' - a(x, t), \tag{1}
\]

\[
\tau \frac{\partial a(x, t)}{\partial t} = bu(x, t) - a(x, t) \tag{2}
\]

posed on a disk \(S\) of finite radius. Spiral waves in this model are naturally defined by the border between low and high states of neural activity.

Dimensionally reduced system of equations can be derived using a recent interface approach [3]. Differentiating the level set \(u(x, t) = h\) along the contour \(\partial B(t)\) allows us to obtain normal velocity, \(c_n \equiv \mathbf{n} \cdot \frac{dr}{dt} = \frac{F(c_n)}{|F(n)|}\), where \(\mathbf{r}\) is a point on domain boundary. Using the Reynold’s transport theorem we find (after dropping transients)

\[
F(z(s, t)) = \int_{0}^{t} dt' \eta(t') \oint_{\partial B(t-t')} ds' w(|r(s, t) - r(s', t')|)z(s', t - t'). \tag{3}
\]

We can now analyse the interface model directly rather than the more computationally expensive space-time model defined by (1) and (2).

References
