Renewal theory approach to the stability analysis of spiking recurrent networks

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The classical approach to the analysis of the stability of integrate-and-fire network of neurons is to apply Fokker-Planck equation for the voltage distribution with boundary conditions. This approach is highly successful for networks with one state variable neurons and static couplings. Nonetheless, it is considerably difficult to extend it to networks of neurons with more than one state variable (e.g. networks with the dynamic synapse or correlated input), due to the boundary conditions that become highly non-trivial. To overcome these problems, we present an alternative approach: instead of studying the evolution of the voltage distribution, we analyze the system by studying how the inter-spike interval distribution evolves. This determines evolution of the firing rate, which determines the feedback input. Thus, self-consistency determines spiking rate in the asynchronous state in the recurrent network. This approch also allows to determine the eigenvalue-equation in the perturbed system. Here, we recover the same eigenvalue equation as the classical Fokker-Planck approach for a network of neurons with static synapses without noisy derive [2]. Furthermore, we show the analysis can easily be extended to networks more state variables. As a computationally interesting example, we illustrate phase transition of a recurrent network of integrate-and-fire neurons with Short-Term-Depression (STD). It worth to mention that an extra state variable in STD, where a slow recovery time-constant goes beyond the inter-event-intervals, leads to the departure of the spiking process from the assumptions of the canonical renewal theory. However, we demonstrate that using our formalism the stability condition of such a system still can be formally analyzed. Here, much insight is gained form our method that activity dependent recurrent couplings contribute to the phase transition of the system in the presence of noisy derive from asynchronous state to collective dynamics of population spikes (i.e. synchronization). Our results here provide a spiking based theory for the previously known phenomena of populations spike [4] achieved by rate approximations of the mean field approximations. The numerical simulations demonstrate a good agreement with the linear stability analysis based on the self-consistent spiking statistics formalism.

The complete account of the method and its detailed analysis is available at [1].

References

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