Travelling Wave and Bump Dynamics in a Spiking Neuronal Network

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As a result of modern imaging technologies, waves and bumps of neuronal activity have been experimentally verified at a variety of spatial scales in the cortex. Spatially localised bumps of activity are known to be involved in mechanisms of orientation tuning in the visual cortex, the rat head direction system, and working memory. In the turtle visual cortex, the presentation of stimuli has been shown to evoke propagating waves of activity. Also, mental processes including sleep and binocular rivalry are characterised through waves, as well as neurological disorders such as epilepsy and migraines.

In this poster we study existence and stability of coarse bumps and travelling waves in a bi-stable spiking neuronal network originally proposed by Laing and Chow [1]. The network consists of a set of N leaky Integrate-and-Fire neurons with a non-local lateral inhibition connectivity function,

$$\frac{dv_i(t)}{dt} = I - v_i(t) + \sum_{j,m} W_{ij}\alpha(t - t_j^m) - \sum_{i,m} \delta(t - t_i^m).$$

Direct numerical simulation show that the model displays a rich repertoire of emergent dynamics including: bumps, multi-bumps and travelling waves with variable number of threshold crossing. We make a continuum assumption and determine analytical solutions of waves in terms of firing times and wave speed, similarly to what was done by Osan et al [2]. The approach we use is semi-analytic, in the sense that we construct analytically well-posed boundary-value problems in the unknown firing times and speed, and then resort to numerical quadrature to compute integrals. We show that the analytical waves match the ones found in the discrete network, in the limit $N \to \infty$. We then study the bifurcation structure of such waves as the strength and timescale of post-synaptic input are varied simultaneously. Stability is analysed by perturbing firing times of the wave and deriving a corresponding Evans function. We provide numerical evidence that waves are robust under extrinsic noise in neuronal input and random neural connections.

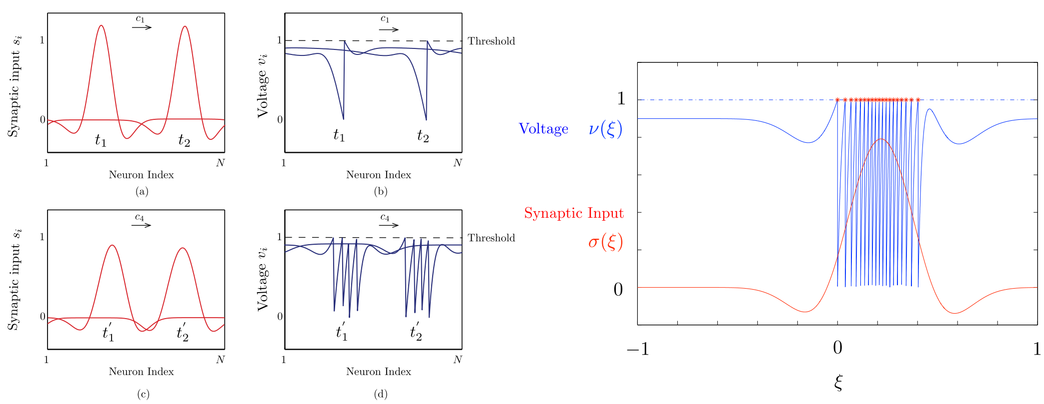

Figure 1: Left: Analytical solution of synaptic input and voltage profile for a single and quad-spike pulse. Right: Solution for pulse with 20 threshold crossings in travelling wave co-ordinates. All waves can be fully depicted by wave speed c and a set of threshold crossing times $T_1, T_2, ..., T_M$

References

[1] C. Laing, C. Chow. Stationary Bumps in Networks of Spiking Neurons, *Neural Computation*, 13, 1473-1494 (2001).

[2] R.Osan, et al. Multiple-Spike Waves in a one-dimensional Integrate-and-Fire Neural Network, *J. Math. Biol*, 48, 243-274 (2004)