Non-stationary synaptic conductances modeled by filtered shot noise processes

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We have investigated key statistical properties of systems that can be described by the filtering of shot noise input through a first-order Ordinary Differential Equation (ODE) with variable coefficients. Such systems give rise to filtered shot noise processes with multiplicative noise. Filtered shot noise processes have proven to be very effective in modelling the evolution of systems exposed to stochastic shot noise sources, and have been applied to a wide variety of fields ranging from electronics through biology [1, 2, 3]. In particular, filtered shot noise processes can model the membrane potential V_m of neurons driven by stochastic input [4, 5], where they are able to capture the non-stationary characteristics of V_m fluctuations in response to pre-synaptic input with variable rate. Previous work has often focused on the stationary regime of shot noise conductances with constant rate, and an exact analytical solution has been obtained for the mean and joint moments of exponential shot noise [6, 7]. However, many neuronal systems evolve in non-stationary regimes driven by shot noise with variable input rate. A typical example is provided by visual system neurons that receive pre-synaptic input with time-varying rate that reflects an evolving visual landscape. Poisson Point Processes (PPP) provide a natural model of random input arrival times that are distributed according to a Poisson law that may vary in time.

The key idea of our approach is to express the filtered process as a transformation of random input arrival times and to apply the properties of PPP transformations to derive its non-stationary statistics. We first identified the causal PPP transformation that corresponds to filtered shot noise with multiplicative noise. We investigated the statistical properties of this transformation to derive the exact analytical solution for the joint cumulants of the filtered process with variable input rate. Excellent agreement with numerical simulations was found for the mean and second order cumulants.

The analytical expressions for the non-stationary cumulants are somewhat complex and may render more challenging further analytical work. We therefore propose an approximation based on a power expansion around the deterministic solution of the system, that is expressed in the central moments of the integrated shot noise process. This approximation yields excellent results for biologically-plausible parameters and in particular for highly non-stationary regimes. These general results were then applied to a simple model of sub-threshold membrane potential V_m fluctuations subject to shot noise conductance with continuously variable rate of pre-synaptic spikes. Excellent agreement with numerical simulations was found for the mean and second order cumulants for both the exact analytical solution and the second order of the central moments expansion. The second order of this approximation is consistent with previously published analytical estimates of mean and standard deviation of stationary V_m [8, 9, 10], and provides an extension to arbitrary (but well-behaved) shot noise kernels.

We show that the time-evolving density of synaptic conductances and membrane potential can be very accurately approximated by the a third order Edgeworth expansion using the non-stationary cumulants. Finally, we use the *Slivnyak-Mecke Theorem* [11] to extend this formalism to multiple independent shot noise inputs, which enables direct applications for neuronal membrane models with different synapse types (such as excitatory and inhibitory synapses). Other extensions include the analytic treatment of correlated shot noise inputs and the statistical effects of afferent network inhomogeneities.

References

[1] SO Rice *Mathematical analysis of random noise*, Bell Systems Tech. J., Volume 24, 1945.

[2] DL Snyder, MI Miller *Random Point Processes in Time and Space*, Springer, 1991.

[3] E Parzen *Stochastic processes,* SIAM, 1999.

[4] AA Verveen, LJ DeFelice Membrane noise, *Progress in biophysics and molecular biology,* 1974.

[5] HC Tuckwell *Introduction to theoretical neurobiology: volume 2, nonlinear and stochastic theories,* Cambridge University Press, 1999.

[6] L Wolff, B Lindner Method to calculate the moments of the membrane voltage in a model neuron driven by multiplicative filtered shot noise, *Physical Review E,* 2008.

[7] L Wolff, B Lindner Mean, variance, and autocorrelation of subthreshold potential fluctuations driven by filtered conductance shot noise, *Neural Computation,* 2010

[8] A Kuhn, A Aertsen, S Rotte Neuronal integration of synaptic input in the fluctuation-driven regime, *The Journal of neuroscience,* 2004.

[9] MJE Richardson, W Gerstner Synaptic shot noise and conductance fluctuations affect the membrane voltage with equal significance, *Neural computation,* 2005.

[10] M Rudolph, A Destexhe An extended analytic expression for the membrane potential distribution of conductance-based synaptic noise, *Neural Computation,* 2005.

[11] IM Slivnyak Some properties of stationary flows of homogeneous random events, *Theory of Probability & Its Applications,* 1962.