First passage times in integrate–and–fire neurons with stochastic thresholds

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We consider a leaky integrate–and–fire neuron with deterministic subthreshold dynamics and a stochastic firing threshold. The formulation of this minimal model is motivated by the experimentally observed widespread variation of neural firing thresholds [1, 2]. We show numerically that the mean first passage time can depend non–monotonically on the noise amplitude. We provide an explanation for this effect by analytically transforming the original model into a first passage time problem for Brownian motion. This transformation also allows for a perturbative calculation of the first passage time histograms. In turn this provides quantitative insights into the mechanisms that lead to the non–monotonic behaviour of the mean first passage time. The perturbation expansion is in excellent agreement with direct numerical simulations. We explain how the effect studied here depends on different noise scalings and how it is related to inverse stochastic resonance [3, 4]. Our modelling framework allows the incorporation of biophysically detailed components into the subthreshold dynamics, rendering our approach a powerful framework that sits between traditional integrate–and–fire models [5] and complex mechanistic descriptions of neural dynamics.

References

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