Roadmaps for neuronal models: individual and networked

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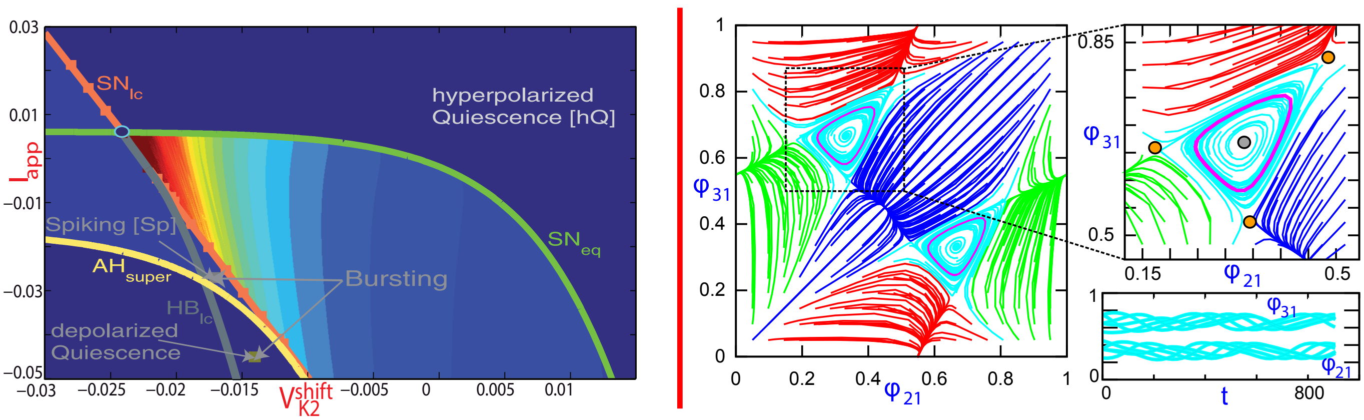
In the talk we present two applications from mathematical neuroscience with detailed biparametric "roadmaps" [1, 2]. Such a roadmap provides an exhaustive information about the dynamics of a single neuron that one must have to build small neuron networks and to study rhythmogenesis in central pattern generators (CPG). In the first application we characterize the systematic changes in the topological structure of chaotic attractors that occur at spike-adding bifurcations in the slow-fast dynamics of neuron models [3]. This phenomenon is detailed in the phenomenological Hindmarsh-Rose (HR) neuron model and a reduced model of the leech heart interneuron (its roadmap is shown in the left panel of the figure below). For the HR model we show that the unstable periodic orbits emerging through spike-adding bifurcations can be described by sequences of the symbolic encoding associated with its complex dynamics [3, 4]. The symbolic description allows us to understand the correlation between the bifurcations and the corresponding metamorphoses of the intrinsic structure of the chaotic attractor of the HR model.



In the second application, we reveal the existence of heteroclinic cycles between saddle fixed points (FP) and invariant circles (IC) (such as shown in the right panel in the figure) in a 3-cell CPG network (leech heart neurons [5]). Such a cycle underlies a robust "jiggling" behavior in bursting synchronization. To study biologically plausible CPG models we employ and further develop novel techniques based on the Poincaré return maps for phase lags between coupled bursters [6]. Using the combination of these techniques we are able to aggregate big data to parametrically continue FPs and ICs of the maps and to fully disclose their bifurcation unfoldings as the network configuration is varied [7].

References

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