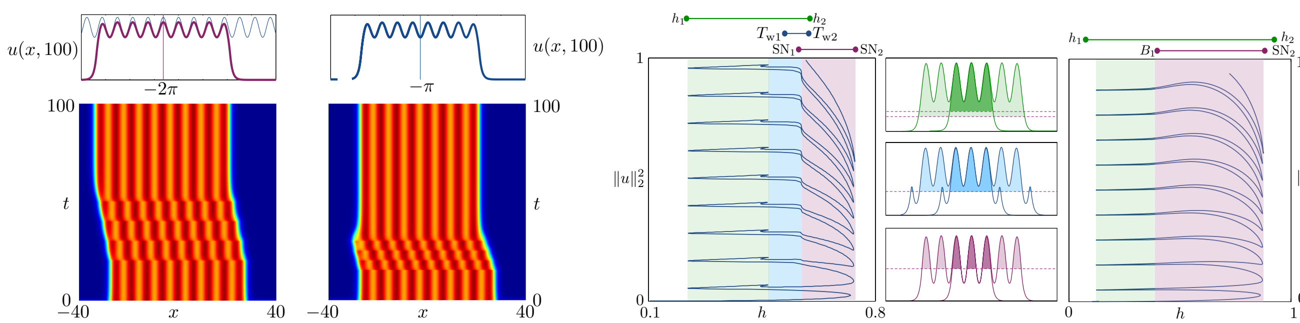
Snakes and ladders in neural field models: analytical and computational tools.

Daniele Avitabile* Helmut Schmidt†

In this talk we will discuss the existence and bifurcation structure of stationary localised solutions to neural field models with spatial heterogeneities. Localised bumps of neural activity are related, for instance, to short term working memory and feature selectivity in the visual cortex. It is known that localised bumps occurring in translation-invariant neural fields may be arranged on snaking branches of solutions (*snake and ladder bifurcation structure*) [5, 6, 7] in a similar fashion to what is found for the Swift–Hohenberg equation and its variants [1]. In this talk we show how such bifurcation structure can be determined analytically in the limit of Heaviside firing rates. As an example we consider a scalar neural field model with spatially-periodic heterogeneities [2]

$$\partial_t u(x,t) = -u(x,t) + \int_{-\infty}^{\infty} W(x,y) f(u(y,t))\,\mathrm{d}y, \qquad W(x,y) = \frac{1}{2} e^{|x-y|} \left[1 + a \cos \frac{y}{\varepsilon} \right] \tag{1}$$

where f is a steep sigmoidal or a Heaviside firing rate function. It is well known that, in the limit of Heaviside firing rate functions, it is possible to construct and infer linear stability of stationary states of neural fields models (see [3, 4] and references therein). We show that, as a consequence, one can compute analytically snaking branches of localised states, together with their parities, thereby gaining a full analytical treatment of the snake and ladder bifurcation structure. In the case of steep sigmoidal firing rates we use numerical continuation and find that the Heaviside calculations predict accurately the structure obtained for smooth sigmoids. Unlike in the standard Swift–Hohenberg case, it is possible to build connections between the trivial state, the fully-periodic state, and a third coexisting periodic state with multiple threshold crossing, giving rise to snaking branches with multiple re-entrances. The Heaviside limit describes them accurately and shows that, in model (1), snaking is a direct consequence of the heterogeneity of the neural tissue. Towards the end of the talk we will discuss numerical computations in neural field models similar to (1), but posed on a plane. Using Discrete Fast Fourier Transforms in combination with Newton-GMRES solvers it is possible to track states and determine stability in an accurate and efficient way. We will present numerical calculations in a simple model of the visual cortex.



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*Presenting author, School of Mathematical Sciences University of Nottingham, Daniele.Avitabile@nottingham.ac.uk.

†College of Engineering, Mathematics and Physical Sciences, University of Exeter, UK, H.Schmidt@exeter.ac.uk.