On the effects of the pinwheel network symmetries on cortical response.

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A recent work is presented, in which the mathematical description of the spontaneous activity of V1 was revisited by combining several important experimental observations including 1) the organization of the visual cortex into a spatially periodic network of hypercolumns structured around pinwheels, 2) the difference between short-range and long-range intra-cortical connections, the first ones being rather isotropic and producing naturally doubly-periodic patterns by Turing mechanisms, the second one being patchy and 3) the fact that the Turing patterns spontaneously produced by the short-range connections and the network of pinwheels have similar periods.

By analyzing the Preferred Orientation (PO) map, we are able to classify all possible singular points of the PO maps (the pinwheels) as having symmetries described by a small subset of the wallpaper groups. We then propose a description of the spontaneous activity of V1 using a classical voltage-based neural field model that features isotropic short-range connectivities modulated by non-isotropic long-range connectivities. A key observation is that, with only short-range connections and because the problem has full translational invariance in this case, a spontaneous doubly-periodic pattern generates a 2-torus in a suitable functional space which persists as a flow-invariant manifold under small perturbations, hence when turning on the long-range connections. Through a complete analysis of the symmetries of the resulting neural field equation and motivated by a numerical investigation of the bifurcations of their solutions, we conclude that the branches of solutions which are stable over an extended set of parameters are those corresponding to patterns with an hexagonal (or nearly hexagonal) symmetry.

The question of which patterns persist when turning on the long-range connections is answered by 1) analyzing the remaining symmetries on the perturbed torus and 2) combining this information with the Poincaré-Hopf theorem to select the possible generic patterns and dynamics that can occur. We have developed a numerical implementation of the theory that has allowed us to produce the patterns of activities predicted by the theory. In particular we generalize the contoured and non-contoured planforms predicted by previous authors [1, 2] and predict the existence of mixed contoured/non-contoured planforms. We also found that these planforms are most likely to be time dependent.

References

[1] GB Ermentrout and JD Cowan. A mathematical theory of visual hallucination patterns, *Biological cybernetics* 1 (2) pp. 1234-1245, 1979.

[2] P.C. Bressloff, J.D. Cowan, M. Golubitsky, P.J. Thomas, and M.C. Wiener. *Geo- metric visual hallucinations, euclidean symmetry and the functional architecture of striate cortex*, *Phil. Trans. R. Soc. Lond. B* 306(1407):299330, 2001.