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Title: Stochastic neural field equations

Abstract :

Neural field equations are used to describe the spatiotemporal evolution of the average activity in a network of synaptically coupled populations of neurons in the continuum limit. This deterministic description is only accurate in the infinite population limit and the actual finite size of the populations causes deviations from the mean field behavior. One way to take into account fluctuations is to add noise to the neural field equations. The resulting class of stochastic neural field equations exhibit a rich phenomenology and it is the main aim of this talk to introduce a complete mathematical framework for the rigorous multiscale analysis of stochastic neural field equations.

Stochastic neural field equations have first been introduced in the paper [2] to study various effects of multiplicative noise on traveling waves. First rigorous results concerning well-posedness of these equations appeared in the paper [3], while dynamical equations for the asymptotic fluctuations in approximating Markov chains have formally been identified in [6].

In our talk we will first rigorously derive stochastic neural field equations with noise terms accounting for finite size effects. These equations are identified by describing the evolution of the activity of finite-size populations by Markov chains and then determining their asymptotic fluctuations. The jump rates are of a different form than considered in the literature so far (see [1]) and lead to qualitatively different results. In particular the fluctuations around stable stationary solutions are of smaller order than previously assumed. We then introduce a complete mathematical framework for the analysis of the resulting stochastic neural field equations.

As first steps of a multiscale analysis, a geometrically motivated decomposition of the stochastic evolution into a randomly moving wave front and fluctuations is derived next. A random ordinary differential equation describing the velocity of the moving wave front can be deduced and the fluctuations around the wave front turn out to be non-Gaussian, even if the driving noise term is a Gaussian process. We will compare our results with the findings on the impact of noise on the position and velocity of the wave front obtained in [2]. Thereby we will summarize recent results on the Lyapunov stability of wave fronts, necessary for our analysis.

The presented geometric approach is in principle applicable to describe the statistics of any macroscopic profile driven by spatially extended noise, like, e.g., wave fronts and pulses in general stochastic reaction diffusion systems

(see [7]).

The talk is partially based on joint work with Jennifer Krüger ([4]) and Eva Lang ([5]).

References

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