Standing and travelling waves in a spherical brain model: the Nunez model revisited

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The Nunez model for the generation of electroencephalogram signals is naturally described as a neural field model on a sphere with space-dependent delays [1] For simplicity, dynamical realisations of this model, formulated either as a damped wave equation or an integro-differential equation, have typically been studied in idealised one dimensional or planar settings. Here we revisit the original Nunez model to specifically address the role of spherical topology on spatio-temporal pattern generation.

Specifically, we consider the neural field with space-dependent delays

$$\frac{\partial u(t,\mathbf{r})}{\partial t} = -u(t,\mathbf{r}) + \iint_{\Omega} w(\mathbf{r}\cdot\mathbf{r}')\, f \circ u\big(t - \tau(\mathbf{r}\cdot\mathbf{r}'), \mathbf{r}'\big)\, d\Omega(\mathbf{r}')$$

where $\Omega = S^2$ is the unit sphere in $\mathbb{R}^3$ and $\mathbf{r}\cdot\mathbf{r}'$ denotes the dot product. Moreover, w and τ respectively denote the synaptic strength and delay as a function of the spatial separation between points. Proceeding with Turing instability analysis, we identify the stability region in parameter space. For each boundary of the stability region, symmetric bifurcation theory gives us a normal form equation, that allows us predict how many and which spatio-temporal patterns arise at these instabilities. Using a center manifold reduction, the critical normal form coefficients are evaluated using the method of sun-star calculus [2].

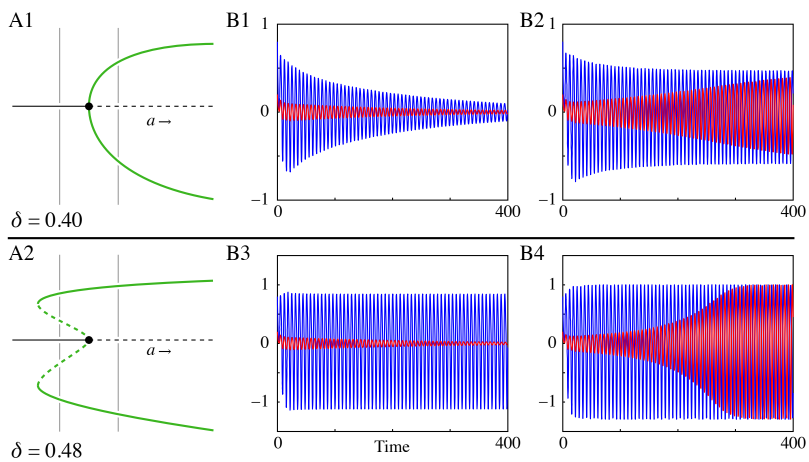

Figure 1: The framework of sun-star calculus predicts a generalized Hopf bifurcation, where the criticality changes from super (A1) to sub (A2). The presence of multistability related to this bifurcation is confirmed by direct numerical simulations: B3 reveals the co-existence of a stable focus (red) and a stable limit cycle (blue)

For example, Fig. 1 shows how theoretical predictions regarding bistability correspond with direct simulations. Interestingly, this bifurcation, which is often seen as a pathway to epileptic seizures [3], was not identified before in the presence of delays. Other types of instabilities we identify are double Hopf bifurcation and bistability between standing and traveling waves.

References

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