A Gauge theory for coupling cortical layers

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We will introduce a model for coupling lateral geniculate nucleus and V1-V2 in terms of a Gauge field theory. The task will be accomplished by writing the action of every layer in terms of covariant derivatives. Particularly the action of the thalamus will code for the particle term and the action of V1-V2 as the field term of a classical gauge field theory. To couple the two elements a classical interaction term will be introduced. In this way, we will obtain an analogue of the classical theory of electromagnetism where both the particle and the fields are the unknowns of the problem. Indeed, we propose a complete Lagrangian, sum of three terms: a particle term, an interaction term and a field term. The Lagrangian will be written in terms of covariant derivatives. This choice allows us to obtain a model that is intrinsic, i.e. independent on the choice of any global or local reference system, exactly as for contemporary physical theories. Once this property is clearly established, for simplicity we express the equations in term of the usual derivatives in the global Cartesian reference system.

The particle term is

$$\mathcal{L}_1 = \int |\nabla\phi - \nabla h|^2 dxdy \tag{1}$$

and is directly inspired by the retinex model of [2]. It performs the differentiation of the visual input h with laplacian of Gaussian receptive profiles and describes the reconstruction ϕ of the image from its existing boundaries. It implements the perceptual invariance with respect to contrast.

The second term describes the interaction between particle and the gauge field $\vec{A}$:

$$\mathcal{L}_2 = \int |\nabla\phi - \vec{A}|^2 dxdy. \tag{2}$$

This term is able to couple the boundaries of the reconstructed image with the field $\vec{A}$, that is unknown and describes both the existing and illusory boundaries. This term $\mathcal{L}_2$ acts on ϕ, and it expresses the reconstruction of the image from the old and new boundaries explaining perceptual figure completion, by keeping contrast invariance properties. At the same time this term acts on the unknown vector field $\vec{A}$, which will be forced to have the direction of $\nabla\phi$. Hence it models both the action of feedback and feedforward connectivity.

Finally we impose a field term modeling the action of V1-V2:

$$\mathcal{L}_3 = \int |\nabla\vec{A}|_g^2 dxdy \tag{3}$$

where the norm is defined with respect to the metric g that is the classical sub-Riemannian metric of the functional architecture of V1-V2 [1]. This term expresses the propagation of existing contours by long range connectivity and allows the creation of subjective boundaries.

A complete Lagrangian is obtained by summing up: $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$, that is invariant by the gauge transformation $h \to h + \quad, \phi \to \phi + \quad, \vec{A} \to \vec{A} + \nabla \quad$.

Calculus of variations yields coupled Euler Lagrange equations:

$$\Delta\phi = \frac{1}{2}(\Delta h + div(\vec{A})), \quad \Delta\vec{A} = -\nabla\phi + \vec{A} \tag{4}$$

that, ones numerically approximated and solved, are able to reproduce a variety of experiments of phenomenology of perception, starting from the modal completion of the celebrated Kanizsa triangle.

The model is extended to take into account cells sensitive to color and allowing to explain the well known phenomenon of neon color spreading.

References

[1] G. Citti, A. Sarti, A Cortical Based Model of Perceptual Completion in the Roto-Translation Space, J. Math. Imaging and Vision, 24 (3), 307-326, 2006. Title of the paper, *Journal* 1 (2) pp. 1234-1245, 2014.

[2] J-M Morel, A. Beln Petro, C. Sbert, A PDE Formalization of Retinex Theory, IEEE Transactions on image processing, 19 (11), 2010.