Stochastic synchronization of neural activity waves
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Summary. We demonstrate that waves in distinct layers of a neuronal network can become phase-locked by common spatiotemporal noise. This phenomenon is demonstrated for stationary bumps, traveling waves, and breathers. A weak noise expansion is used to derive an effective equation for the position of the wave in each layer, yielding a stochastic differential equation with multiplicative noise. Stability of the synchronous state is characterized by a Lyapunov exponent, which we can compute analytically from the reduced system. Our results extend previous work on limit-cycle oscillators, showing common noise can synchronize waves in a broad class of models.

Further Details. Synchronization of neural activity waves across distinct network locations can be important in several behavioral and sensory contexts. Waves of activity in the visual cortex ensure a portion of the network is on limit-cycle oscillators, showing common noise can synchronize waves in a broad class of models. Simulations show that waves forced by common spatiotemporal noise d

\frac{du_j(x,t)}{dt} = \left[-u_j(x,t) + \int_{-\pi}^{\pi} w(x-y)f(u(y,t))dy\right] dt + \varepsilon dW(x,t), \quad j = 1, 2, \quad (1)

where \( u_j(x,t) \) is population activity of population \( j \), \( w(x-y) \) describes synaptic connectivity, and \( f(u) \) is a firing rate nonlinearity. Small amplitude (\( \varepsilon \ll 1 \)) spatiotemporal noise \( dW(x,t) \) is white in time and has spatial correlations \( C(x-y) \). We demonstrate noise-induced synchronizations of bumps, \( u_j \approx U_j(x + \Delta_j(t)) \), for a single realization of (1) in Fig. 1A. Noise perturbations cause each bump to wander diffusively about the spatial domain. Once both bumps’ positions \( \Delta_1 \) and \( \Delta_2 \) meet, they are phase-locked indefinitely. Using perturbation theory, we can derive effective Langevin equations for the bumps’ positions \( d\Delta_j = \varepsilon dW_j(\Delta_j, t) \), driven by multiplicative noise \( W(\Delta, t) = \sum_{k=1}^{\infty} a_k \cos(k\Delta) X_k + b_k \sin(k\Delta) Y_k \). The phase difference \( \phi = \Delta_1 - \Delta_2 \) will have geometric mean \( \phi_0 e^{\lambda t} \), and we can approximate the Lyapunov exponent \( \lambda \) as shown in Fig. 1B, C [4]. We will discuss similar results can be derived for the case of traveling waves and breathers.

![Figure 1: Common noise-induced phase-locking of bumps evolving in two uncoupled stochastic neural fields, Eq. (1) with \( w(x) = \cos(x) \), and \( C(x) = \cos(x) \). (A) Noise drives bump positions \( \Delta_1(t) \) (solid line) and \( \Delta_2(t) \) (dashed line) to the absorbing state \( \Delta_1(t) = \Delta_2(t) \). Here, \( f(u) = H(u - 0.5) \) and \( \varepsilon = 0.01 \). (B) Realizations of the phase-difference \( \phi = \Delta_1 - \Delta_2 \) (thin lines) compared to the prediction \( \phi(t) = \phi(0)e^{\lambda t} \) (thick line). (C) Numerically calculated Lyapunov exponents \( \lambda \) (circles) are well approximated by theory (solid line).](image)