Stochastic synchronization of neural activity waves

Zachary P. Kilpatrick, University of Houston, zpkilpat@math.uh.edu

Oral Presentation Submission

Summary. We demonstrate that waves in distinct layers of a neuronal network can become phase-locked by common spatiotemporal noise. This phenomenon is demonstrated for stationary bumps, traveling waves, and breathers. A weak noise expansion is used to derive an effective equation for the position of the wave in each layer, yielding a stochastic differential equation with multiplicative noise. Stability of the synchronous state is characterized by a Lyapunov exponent, which we can compute analytically from the reduced system. Our results extend previous work on limit-cycle oscillators, showing common noise can synchronize waves in a broad class of models.

Further Details. Synchronization of neural activity waves across distinct network locations can be important in several behavioral and sensory contexts. Waves of activity in the visual cortex ensure a portion of the network is always maximally sensitive to input [1]. Our findings implicate an potential role for large-scale correlations in nervous system fluctuations. Our analysis extends previous work which showed common noise can synchronize the phases of distinct limit cycle oscillators [2]. The synchronous state is absorbing when each oscillator receives identical noise, and stability can be determined by computing an associated Lyapunov exponent. We show these principles can be applied to waves forced by common spatiotemporal noise.

We analyze the stochastic dynamics of waves in a pair of uncoupled neural field models driven by common noise. Neural fields are integrodifferential equations whose integral term describes the connectivity of a neuronal network. Recent studies have considered stochastic versions of neural field equations, formulated as a Langevin equation [3]:

$$\mathrm{d}u_j(x,t) = \left[-u_j(x,t) + \int_{-\pi}^{\pi} w(x-y) f(u(y,t)) \mathrm{d}y \right] \mathrm{d}t + \varepsilon \mathrm{d}W(x,t), \qquad j = 1, 2, \tag{1}$$

where $u_j(x,t)$ is population activity of population, $w(x-y)$ describes synaptic connectivity, and $f(u)$ is a firing rate nonlinearity. Small amplitude ($\varepsilon \ll 1$) spatiotemporal noise $\mathrm{d}W(x,t)$ is white in time and has spatial correlations $C(x-y)$. We demonstrate noise-induced synchronization of bumps, $u_j \approx U_j(x + \Delta_j(t))$, for a single realization of (1) in Fig. 1**A**. Noise perturbations cause each bump to wander diffusively about the spatial domain. Once both bumps' position Δ_1 and Δ_2 meet, they are phase-locked indefinitely. Using perturbation theory, we can derive effective Langevin equations for the bumps' positions $\mathrm{d}\Delta_j = \varepsilon \mathrm{d}\mathcal{W}(\Delta_j, t)$, driven by multiplicative noise $\mathcal{W}(\Delta, t) = \sum_{k=1}^{\infty} a_k \cos(k\Delta) X_k + b_k \sin(k\Delta) Y_k$. The phase difference $\phi = \Delta_1 - \Delta_2$ will have geometric mean $\phi_0 \mathrm{e}^{\lambda t}$, and we can approximate the Lyapunov exponent $\lambda = -\varepsilon^2 \sum_{k=1}^{\infty} \left[a_k^2 + b_k^2 \right]$, as shown in Fig. 1**B**,**C** [4]. We will discuss similar results can be derived for the case of traveling waves and breathers.

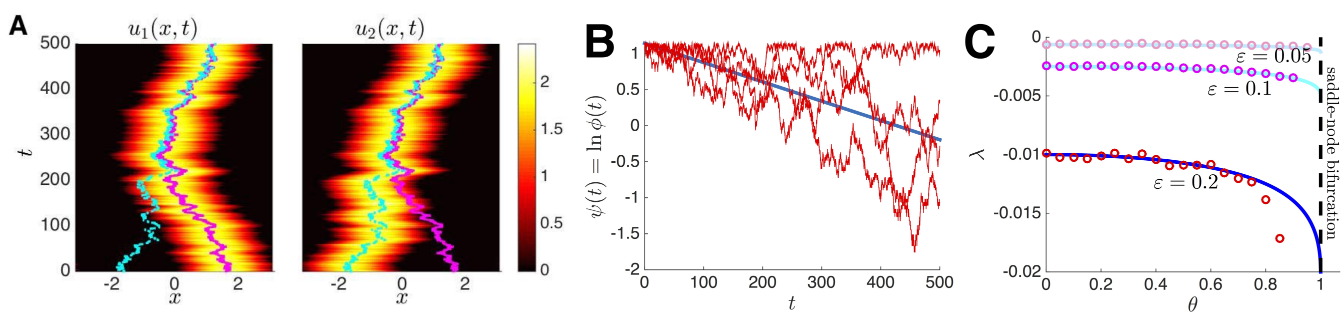

Figure 1: Common noise-induced phase-locking of bumps evolving in two uncoupled stochastic neural fields, Eq. (1) with $w(x) = \cos(x)$, and $C(x) = \cos(x)$. (**A**) Noise drives bump positions $\Delta_1(t)$ (solid line) and $\Delta_2(t)$ (dashed line) to the absorbing state $\Delta_1(t) = \Delta_2(t)$. Here, $f(u) = H(u - 0.5)$ and $\varepsilon = 0.01$. (**B**) Realizations of the phase-difference $\phi = \Delta_1 - \Delta_2$ (thin lines) compared to the prediction $\phi(t) = \phi(0)\mathrm{e}^{\lambda t}$ (thick line). (**C**) Numerically calculated Lyapunov exponents λ (circles) are well approximated by theory (solid line).

[1] G.B. Ermentrout, D. Kleinfeld. Traveling electrical waves in cortex: insights from phase dynamics and speculation on a computational role, *Neuron* 29, pp. 33-44, 2001.

[2] J-n. Teramae, D. Tanaka. Robustness of the noise-induced phase synchronization in a general class of limit cycle oscillators, *Phys. Rev. Lett.* 93, 204103 2004.

[3] P.C. Bressloff. Spatiotemporal dynamics of continuum neural fields, *J. Phys. A* 45, 033001, 2012.

[4] Z.P. Kilpatrick. Stochastic synchronization of neural activity waves, *arXiv* 1412.3889, 2014.