Topological analysis detects intrinsic geometric structure in neural correlations

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Neuroscience data often comes in the form of a matrix W_{ij} (such as a matrix of pairwise correlations or synaptic weights) whose entries are likely to be measured in "unnatural" units that obscure the underlying structure. In other words, W is of the form

$$W_{ij} = f(D_{ij}) + \text{noise},$$

where $f\colon \mathbb{R} \to \mathbb{R}$ is an unknown – but *monotone* – function, and the "hidden" structure in D may be of primary interest. For example, the pairwise correlations W_{ij} between neurons are often believed to reflect distances in an underlying "coding space"; in this case, D is an (unknown) distance matrix and f is an (unknown) monotone decreasing function that transforms distances into correlation strengths.

Detecting meaningful structure in neural activity and connectivity data is challenging in the presence of such hidden nonlinearities, because traditional eigenvalue-based methods in this situation are often misleading. Nevertheless we have found that if f is monotone, key structural properties of D can be reliably inferred from W using tools from topological data analysis. This is because these tools measure structure that is invariant under monotone transformations. These features are encoded in the *order complex*, a combinatorial object that keeps track of the ordering of matrix entries. We found that topological invariants of the order complex, called *Betti curves*, can be used to distinguish random from non-random structure, and also provide reliable signatures of geometric organization in the presence of an unknown nonlinearity. Unlike previous instances of topological data analysis (see e.g. [1]), our method relies on the *statistical properties* of cycles, as captured by Betti curves and persistence lifetime distributions, and is used as a generic tool for matrix analysis, rather than the analysis of point cloud data.

Using simultaneous recordings of many neurons in behaving rats we found that geometric organization of hippocampal place cell activity – a prerequisite for the existence of spatial receptive fields – can be detected from pairwise correlations alone, without any *a priori* knowledge about the nature of the receptive fields and/or the spatial stimuli. Using simulated data from a model, we confirmed that such geometric structure would be observed as a result of realistic place fields, but would not arise from non-geometric ("scrambled") place fields. Remarkably, we also found that (recorded) hippocampal activity exhibits geometric signatures not only during spatial navigation, but also during non-spatial behaviors such as wheel running and REM sleep. At finer timescales, however, our method detected clear differences in the pattern of correlations during spatial versus non-spatial behaviors. We suggest that clique topology is a powerful new tool for matrix analysis, and one that is especially useful in Neuroscience settings, where the relationship between observed quantities and more meaningful variables is often monotonic, but otherwise incompletely understood.

References

[1] C. Curto, V. Itskov. *Cell groups reveal structure of stimulus space.* PLoS Computational Biology, Vol. 4(10), 2008.

[2] C. Giusti, E. Pastalkova, C. Curto*, V. Itskov * (**equal last authors*). Clique topology reveals intrinsic geometric structure in neural correlations. *Submitted*, 2015.