Traveling pulses in some nonlocal FitzHugh-Nagumo equations

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In this oral contribution, I would like to discuss some recent developments in the study of the FitzHugh-Nagumo equations appearing in the modeling of the initiation and propagation of action potentials along axons of neurons. These equations, that can be seen as a simplified version of the well-known Hodgkin-Huxley equations, have served in the past few decades as a prototypical example of a slow-fast system. Namely, these equations are of the form

$$\frac{\partial u}{\partial t} = \mathcal{L} \cdot u + f(u) - v, \tag{1a}$$

$$\frac{\partial v}{\partial t} = \epsilon(u - \gamma v), \tag{1b}$$

where f can be chosen of cubic type (for example $f(u) = u(1-u)(u-a)$, with $0 < a < 1$), $\gamma > 0$ and $0 < \epsilon \ll 1$. In the local case, the operator $\mathcal{L}$ is the usual Laplacian operator Δ and the associated traveling wave problem can be cast as a slow-fast system. Some recent studies [1, 2] have dealt with nonlocal operators of the form $\mathcal{L} = \delta_{-1} + \delta_1 - 2\delta_0$, where δ stands for a Dirac mass, and of the form $\mathcal{L} = -\delta_0 + \mathcal{K}*$, where $\mathcal{K}*$ represents the convolution with the kernel $\mathcal{K}$. After discussing the main results obtained in collaboration with Arnd Scheel on the existence of traveling pulses for system (2) with the nonlocal operator $\mathcal{L} = -\delta_0 + \mathcal{K}*$, I would also like to illustrate some future challenges encountered when considering Dirichlet to Neumann operators for $\mathcal{L}$. This type of operator arises naturally in the original modeling of action potentials along axons using cable theory. In that case, in a first (crude) approximation, the operator $\mathcal{L}$ can be understood as a Fractional Laplacian $-(-\Delta)^s$, with $0 < s < 1$, which suggests the study of nonlocal slow-fast system of the form

$$\frac{\partial u}{\partial t} = -(-\Delta)^s u + f(u) - v, \tag{2a}$$

$$\frac{\partial v}{\partial t} = \epsilon(u - \gamma v). \tag{2b}$$

I will present some first results for this type of problem.

References

[1] H.J. Hupkes and B. Sandstede, *Travelling pulses for the discrete FitzHugh-Nagumo system.*, SIAM J. Appl. Dyn. Syst., **9** (2010), pp. 827–882.

[2] G. Faye and A. Scheel, *Existence of pulses in excitable media with nonlocal coupling*, Adv. Math., **230** (2015), pp. 400–456.