Complex multiple timescale dynamics in a periodically forced Wendling-Chauvel neural mass model

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The Jansen-Rit model is a neural mass model designed to understand the activity of populations of neurons during healthy and pathological brain activity, in particular during epileptic seizure. The Wendling-Chauvel model is a modification of the previous one, and its bifurcation analysis has been performed in [6]. In particular, two main parameters, J (average number of synapses between populations) and P (external input) have been varied across suitable interval where stationary and periodic regimes were analysed in this region of parameter space. According to the value of J, one-parameter bifurcation diagrams in P display different scenarios of transition from stationary to small-amplitude periodic states (α-rhythm) to large-amplitude periodic states (epileptic states). The results are presented in figure 17 of [6]. In certain bands of values of parameter J, bistability is found, either between stationary states and large-amplitude (epileptic) periodic states (zone B) or between small-amplitude (α rhythm) and large-amplitude (epileptic) periodic states (zone C). The idea that we wish to develop, and for which we present preliminary numerical results in this talk, is to allow the external input P to vary slowly so as to create *bursting oscillations* with transitions between the different stable states (stationary and/or periodic) observed in the original model [6]. Henceforth, we highlight possible *critical transition* between healthy and pathological states not due to bifurcation strictly speaking but due to *dynamic bifurcation* and, therefore, to the presence of multiple timescales in the extended system. The strategy of considering slowly varying parameters and resulting dynamic bifurcation is quite standard in singular perturbation theory, and it has proved useful in neuroscience to provide a mathematical framework to understand, in a deterministic context, complex neuronal oscillations, in particular *bursting* [5] and *Mixed-Mode Oscillations (MMOs)* [2]. The former case corresponds, loosely speaking, to a slow passage through the bistable regime of a given oscillator, whereas the latter one corresponds to a slow passage through the canard [4] regime of a slow-fast oscillator. The difficulty here is to find a suitable dynamics for P, that is, such that P goes back and forth through the bistable periodic regime of the Wendling-Chauvel model in zone B. What is more, we wish to create interesting slow-fast dynamics in the resulting extended system, involving canard segment (that is, segment spending an $O(1)$-time close to repelling slow manifolds) close to repelling branches of limit cycles. This other form of canard phenomenon, known as *torus canard*, has been more recently investigated, in the context of neuronal bursters [3, 1]. We include an additional slow variable which forms an oscillator in combination with P. The dynamics of P and Q are given by a slow-fast oscillator that is coupled unidirectionally to the original system, simply forcing it in a feed-forward way through P. This is equivalent to assuming that P is a non-constant forcing to the original system. In fact, this construction of an extended Wendling-Chauvel model, and the simulations showcasing canard solutions, tell us that, locally, torus canard dynamics is essentially the result of a *dynamic saddle-node bifurcation of limit cycles*. Thus, any slow dynamics on P would create at least transiently such complex oscillations.

Depending on the amplitude of the forcing oscillations, we can observe a transition to bursting through torus canards. Indeed, when simulating the resulting system in one parameter regime for which the WC model displays bistability, we find robust torus canard cycles, both *with head* and *without head*, within a small parameter variation. This is the parameter region where the WC model displays bistability between small-amplitude (α-rhythm) and large-amplitude (epileptic) limit cycles. We find a headless torus canard cycle where the complex oscillatory solution shows amplitude modulation around the family of small-amplitude limit cycles of the original model. By slightly modifying the constant term in the P-equation, we also find a torus canard with head where the complex oscillatory solution alternates small-amplitude bursts (close to the family of unstable small-amplitude limit cycles of the WC model) and large-amplitude bursts (close to the family of stable large-amplitude limit cycles of the WC model).

References

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