Neurons in mean-field interaction.

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We are interested in a networked integrate-and-fire model describing an infinite population of neurons which interact with one another through their common statistical distribution. The interaction is of the self-excitatory type as, at any time, the potential of a neuron increases when some of the others fire: precisely, the kick it receives is proportional to the instantaneous proportion of firing neurons at the same time.

We first investigate the well-posedness of the model ([1]). From a mathematical point of view, the coefficient of proportionality is of great importance as the resulting system is known to blow-up as this becomes large. In the current paper, we focus on the complementary regime and prove that existence and uniqueness hold for all time when the coefficient of proportionality is small enough.

We then discuss the construction and approximation of solutions to this model ([2]). The instantaneous nature of the excitation makes the system singular and prevents the application of standard results from the literature. Making use of the Skorohod M1 topology, we prove that, for the right notion of a 'physical' solution, the nonlinear equation can be approximated either by a finite particle system or by a delayed equation. As a by-product, we obtain the existence of 'synchronized' solutions, for which a macroscopic proportion of neurons may spike at the same time.

References

[1] F. Delarue, J. Inglis, S. Rubenthaler, E. Tanré, Global solvability of a networked integrate-and-fire model of McKean-Vlasov type, *to appear in Annals of Applied Probability.*

[2] F. Delarue, J. Inglis, S. Rubenthaler, E. Tanré, Particle systems with a singular mean-field self-excitation, *to appear in Stochastic Processes and their Applications,.*